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SIMULATION OF A VISCOUS FLOW PAST A THREE DIMENSIONAL OBSTACLE USING VORTEX PARTICLES

Piotr Duszyński, Marta Poćwierz, Andrzej Styczek and Jacek Szumbarski
Department of Aerodynamics, Warsaw University of Technology
24 Nowowiejska Str, 00-665 Warsaw, POLAND

Summary The new formulation of the stochastic vortex particle method has been presented. Main elements of the algorithms: the construction of the particles, governing equations, stretching modeling and boundary condition enforcement are described. The test case is the unsteady flow past a spherical body. Sample results concerning patterns in velocity and vorticity fields, streamlines, pressure and aerodynamic forces are presented.

Governing equations

The problem of determination of the a viscous incompressible flow can be formulated in terms of velocity and vorticity fields. The equation of the vorticity transport (the Helmholtz equation) contains the advection and diffusion terms, and also the source term describing the vorticity stretching. One can split this equation (using the Lee-Trotter theorem) into the advection/diffusion equation without a source term and the equation describing vorticity dynamics due to pure stretching. The first one can be interpreted as a Fokker-Planck-Kolmogorow (FPK) equation. Thus, the advection/diffusion part of the vorticity evolution can be simulated by means of stochastic tools. The vorticity field is discretized by a large ensemble of (artificial) vortex particles. Motion of these particles is governed by Ito stochastic differential equations corresponding to FPK equation, i.e., they move with a local fluid velocity and perform a random motion modeling viscous diffusion of the vorticity.

Vortex particles

The construction of a vortex particle is an important element of the method. The velocity field induces by a particle is determined by the rotation operator applied to the product of vector time-dependent quantity (the ”vorticity charge” of the particle) and spherically symmetric scalar function. The vorticity charges change in time accordingly to the stretching equation. It should be emphasized that both velocity and vorticity fields induced by such particles are strictly divergence-free.

Boundary conditions

The no-slip boundary condition is imposed on the surface of an immersed body. To enforce this condition, the surface vorticity layer should be continuously generated. Such layer can be approximated by a number of newly created vortex particles covering tightly the surface of the obstacle. An instantaneous velocity field can be decomposed into the several parts: the one induced by the vortex particles created in the past time steps, the other induced by the new vortex particles added to the vorticity layer and the potential components (gradients of harmonic potentials), which are necessary to cancel out the normal component of the velocity at the material boundary. The velocity field described above is divergence-free and satisfies a far-field boundary condition and impermeability condition on the obstacle's surface. The cancellation of the tangent velocity on this surface is achieved by an appropriate selection of the vectors vorticity charges of the new particles. It turns out that these vectors can be assumed parallel to the body's surface. Once the new vortex particles are fully determined, they become the ”old” particles and begin to move in the flow domain as described earlier.

Global vorticity constrains

The analysis of the Helmholtz equation reveals an important constrain imposed on the vorticity field: the total vorticity charge of the vortex particles has to be steady in time. If the flow starts from rest the total charge remains zero. In principle, the proposed method of the vortex particles obeys this restriction automatically. However, in practical computations it is not necessarily the case because of inevitable numerical errors. In order to avoid a possible instability, an artificial vortex particle located outside the flow domain (in the interior of the obstacle) is introduced, which compensates error in the vorticity balance.

Pressure field

The pressure field is determined afterwards on the basis of previously calculated velocity and vorticity fields. In contrast to 2D flows (where the flow domain is not simply connected), the pressure problem is well-defined and posses the unique physically meaningful solution without any additional restriction.

Numerical computations

The numerical computations have been carried out for the spherical body. The results have been obtained for several Reynolds numbers. Instantaneous patterns in the velocity and vorticity fields have been obtained. The complexity and shape sensitivity of the instantaneous streamlines originating from closely located points has been demonstrated (see Figure below), which gives idea about the mixing intensity in the wake region. The numerically calculated aerodynamic

reactions do not differ much from the experimental measurements.
The key problem of the practical calculation is the numerical cost of the induced velocity evaluation. For 3D problem, fast summation methods (e.g. Greengard-Rokhlin algorithm) do not work as efficiently as in 2D case. Other tricks like using indirect velocity evaluation (the far-field induced velocity is calculated in the nodes of an auxiliary grid(s) and then interpolated) might help, but various artifacts can be expected. This is why the current implementation of the method relies on direct "each-to-each" computations of the induced velocity. Future improvement of this part of the algorithm will significantly increase its practical value.