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WAVE PROPAGATION IN HIGH POROSITY BONES - A CELLULAR MODEL

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Summary Experimental ultrasonic studies for high porosity cancellous bone give the results which can not be properly described in the wide frequency range (especially for high frequencies) using two phase macrocontinual models (Biot's or Schoenberg model). Hence, a cellular model for propagation of ultrasonic waves in such media is proposed. The sensivity analysis of frequency dependent phase velocities and attenuations of both longitudinal ultrasonic waves is presented in order to extract physical properties and structural parameters of the cellular model which mostly influence wave parameters. The parameters calculated from the proposed model are compared with the data obtained from Biot's theory. It was shown that only for the long wave range (low frequencies) the predictions of both considered models are close to each other and that the results obtained from cellular model are in good qualitative agreement with the experimental data received from ultrasonic studies.

INTRODUCTION

Ultrasonic methods are currently well known and useful tools for non-destructive and non-invasive studies of properties of porous materials, both saturated or dry. An essential component of such studies applied for the complex materials is a mathematical model, which needs to be used for description of wave propagation in the materials and for extracting their mechanical and structural properties from wave parameters (phase velocity, attenuation coefficient and backscattering coefficient). While the macroscopic Biot's model is well verified for many lower porosity materials, in the case of higher porosity materials such as cancellous bones (porosity of cancellous bone is 80 - 90% or even higher) the model is less appropriate. Ultrasonic studies for cancellous bone, [4, 5, 7, 8, 9], show existence of two longitudinal modes, but the properties of the measured waves do not fit the data derived from the existing two phase macroscopic models (e.g. positive dispersion of phase velocity and higher attenuation of the slow wave than attenuation of the fast wave) when the wavelength is comparable with the characteristic pore size [8, 9]. One of the possible sources of the divergence between the model and experimental observations is the fact that the validity of the macroscopic model requires much longer waves than the size of inhomogeneity.
As the result appropriate for studies of short ultrasonic waves (when the averaged wavelength is comparable with or shorter than the pore size) model is needed. This paper develops cellular model of material for propagation of ultrasonic waves in cancellous bones propagating in the main direction of trabecular network.

MODELLING

The model of cellular material as composition of periodic system of two cells along with the exact solution for propagation of plane harmonic waves in each cell, [1, 6], (tube filled with fluid and layered cell composed of solid and fluid) is used. It is assumed that the solid material is elastic and isotropic while fluid is treated as the newtonian. Periodicity of the model allows to limit our analysis to a single cell, [1]. Considering propagation of plane harmonic wave independently for fluid (layer or channel filled with fluid) and solid (layer or tube) the displacements and stresses in the fluid and solid (solutions) are found. In order to receive the characteristic equation the boundary and matching conditions for the cell, outer boundary and interface between the fluid and solid, need to be satisfied. The matching conditions are: the equality of the normal and tangential stresses in the solid and fluid and the continuity of particle velocities normal and tangential to the interface. The outer cell boundary conditions are: the vanishing tangential stresses and normal displacement in the solid, [9]. The received characteristic equation is nonlinear complex algebraic equation for unknown wave number k_z which is solved numerically in order to find the propagation velocity ($V(f)$) and attenuation coefficient ($\alpha(f)$) of the plane harmonic waves as a functions of frequency (f) according to the equations:

$$V(f) = \frac{2\pi f}{Re(k_z)}, \quad \alpha(f) = Im(k_z) \; . \tag{1}$$

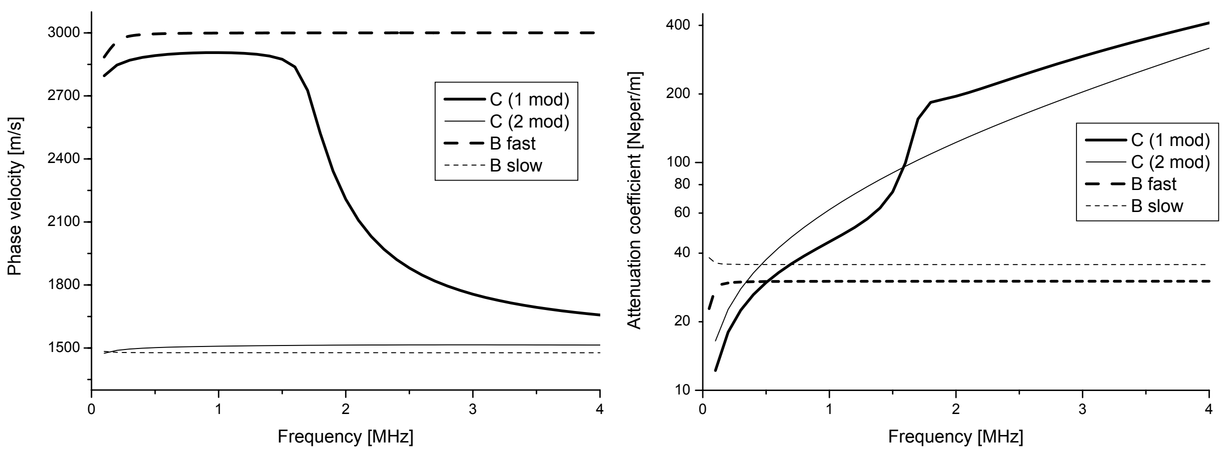
In order to evaluate the role of physical properties of phases and structural parameters of the model the sensivity analysis of frequency dependence of phase velocities and attenuations of both longitudinal US waves was performed. It was shown the wave parameters are practically insensitive to changes of porosity for long waves and the phase densities for the whole frequency range, while the other parameters: fluid viscosity, speed of sound in fluid, shear and longitudinal wave velocity in solid phase and characteristic size of inhomogeneity strongly influence both the phase velocity and attenuation.

It is also interesting to notice that the cellular model predicts infinite number of wave modes, but only two of them exist in the whole frequency range. Additional modes appear at some characteristic frequencies f_n, which satisfy the relation:

$$f_n = \frac{n c_f \pi}{l_f} \sqrt{\frac{c_{fn}^2}{c_{fn}^2 - c_f^2}}\,, \tag{2}$$

where c_f is speed of sound in fluid, c_{fn} - phase velocity at frequency f_n, n- integer number which correspond to consecutive wave modes and l_f is the characteristic size of inhomogeneity in the model (thickness of the layer or channel diameter). Since each frequency f_n is related to the size of channel or cell, the formula (2) can be used in the evaluation of averaged characteristic size of inhomogeneity and its changes during some processes (in the case of cancellous bone - during osteoporosis).

In Fig. 1 the theoretical predictions obtained from the cellular model are compared with results from the macrocontinual (Biot's) model (see Fig.1). It is worth noticing that both considered models predict more than one longitudinal wave modes. The results from the cellular model show strong negative dispersion of the phase velocity of fast wave, while the velocity of slow wave is almost constant for the whole considered frequency range (see Fig. 1_a). In the case of Biot's model velocities of both wave modes (fast and slow) change insignificantly with frequency. For the long waves



(a) (b)

Figure 1. Comparison of phase velocity (a) and attenuation coefficient (b) from the macrocontinual Biot's (B) and cellular model (C) - numerical calculations

(low frequency) wave velocities derived from the cellular model are close to that obtained from Biot's theory. When the frequency increases (for shorter wavelengths) the discrepancy between two models rises, particularly when the fast wave is considered.

Attenuation coefficients (see Fig.1_b) derived from of both models show essentially different qualitative and quantitative behaviour. In particular attenuations from the Biot's model for the whole frequency range have almost constant values. The parameter from cellular model strongly increases as a function of frequency. It is also interesting to stress that accordingly to the cellular model, from some characteristic frequency ($\sim$ 1.7MHz), independently of the porosity, attenuation of the slow wave is lower than that for the fast one, while in terms of Biot's model such behaviour is observed only for high porosities (over 85%).

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