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ON THE PROPAGATION OF SOLITARY WAVES IN MICROSTRUCTURED SOLIDS

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Summary In the present paper a Korteweg–de Vries (KdV) type evolution equation, including the third- and fifth order dispersion and the fourth order nonlinearity is used for modelling the wave propagation in microstructured solids. The model equation is solved numerically under localised initial conditions. Possible solution types are introduced and discussed.

INTRODUCTION

Wave propagation in microstructured media is essentially influenced by nonlinear and dispersive effects. The simplest model governing these two effects results in the celebrated KdV equation $u_t + uu_x + du_{3x} = 0$. However, studies of microstructured materials have shown that higher-order dispersive effects together with higher-order nonlinear effects can give rise to dramatic changes in the behaviour of emerging waves. In the present paper a KdV type evolution equation, including the third- and the fifth order dispersive and the fourth order nonlinear terms,

$$u_t + [P(u)]_x + du_{3x} + bu_{5x} = 0, \qquad P(u) = -0.5u^2 + u^4, \tag{1}$$

is used for modelling the 1D longitudinal wave propagation in microstructured solids. Here u is the excitation, t the time coordinate, x the space coordinate, d and b the third- and the fifth-order dispersion parameters, respectively and elastic potential $P(u)$ introduces the quartic nonlinearity. Logarithmic dispersion parameters $d_l = \log d$ and $b_l = \log |b|$ are used instead of d and b for analysis herein after. The sources of higher order effects can be dislocations in the crystal structure of martensitic-austenitic shape-memory alloys [1, 2].

The character of dispersion depends on the signs of parameters d and b: for $db < 0$ one has normal dispersion, however for $db > 0$ the dispersion is normal for some wavenumbers and anomalous for others [3]. In the present paper the case $d > 0$ and $b < 0$ is considered.

THE PROBLEM AND GOALS.

In [3, 4] we have shown that in the case of harmonic initial conditions the equation (1) admits soliton type solutions without reference to the character of dispersion (normal or anomalous). In the present paper the model equation (1) is solved numerically under localised initial conditions

$$u(x, 0) = A\ \mathrm{sech}^2 \frac{x}{\Delta}, \qquad \Delta = \sqrt{12d/A}. \tag{2}$$

The initial solitary wave (2) corresponds to the analytical solution of the KdV equation. Therefore in systems governed by the KdV equation such solitary waves are called solitons , i.e., they propagate with constant speed and amplitude and interact elastically (they restore their amplitude and speed after the interaction).

Our main goal here is to answer the following questions: (i) whether or not solitary waves (2) can propagate in media described by the equation (1) with constant speed and amplitude and (ii) how do such solitary waves interact in such a media. In the other words, do the solitary waves (2) behave like solitons in media where the higher order effects, governed by the equation (1), are of importance.

RESULTS AND DISCUSSION

Numerical solutions are found in the domain of logarithmic dispersion parameters $0.8 \leq d_l \leq 2.4$ and $1.2 \leq b_l \leq 4.8$. For numerical integration the discrete Fourier transform (DFT) based pseudospectral method [5] and periodic initial conditions

$$u(x, t) = u(x + 4n\pi),\ n = \pm 1, \pm 2, \ldots \tag{3}$$

are used. The length of the space period is chosen 4π in order to separate the initial localised waves.

Solution types

Making use of the numerical results three solution types can be detected. The type of solution depends essentially on the value of the amplitude A of the initial localised wave (2).

First type. Small amplitude initial localised waves decay to a chaotic wave-train (Fig. 1).

Second type. If the amplitude A exceeds a certain value $A_1(d_l, b_l)$ then a wave-train, having regular behaviour in time, forms (Fig. 2). In this case one can discuss about the solitonic character of the solution, recurrence and superrecurrence phenomena.

Third type. If the amplitude has values over a certain critical value $A^*(d_l, b_l)$ then the initial excitation propagates with minimal disturbances, i.e., its speed and amplitude changes by a small extent only during the propagation. The higher the amplitude A, the faster the solitary wave propagating to the right (Figs. 3 and 4). Unfortunately there does not exist such a direct relation between the amplitude and speed like in the KdV case. From the other hand, the higher the amplitude, the more distinctive the left-going small amplitude radiation (Fig. 4).

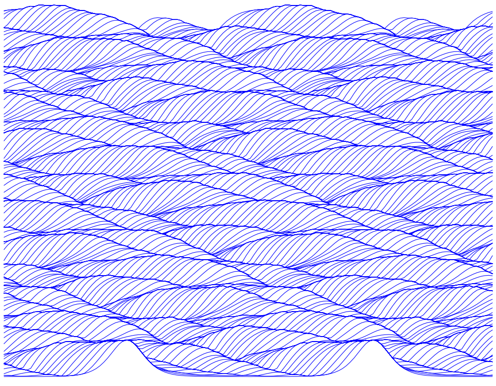

Figure 1. First solution type: time slice plot over two space periods ($d_l = 0.8$, $b_l = 2.0$, $A = 1.5$, $0 \leq t \leq 300$)

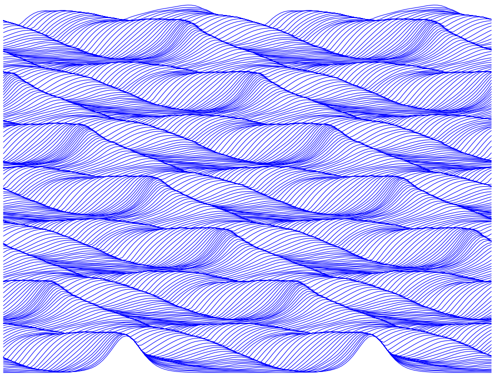

Figure 2. Second solution type: time slice plot over two space periods ($d_l = 0.8$, $b_l = 2.0$, $A = 1.75$, $0 \leq t \leq 300$)

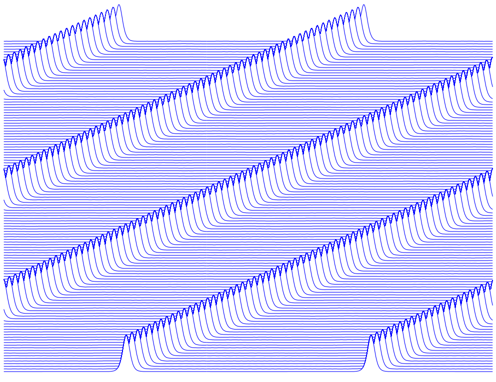

Figure 3. Third solution type: time slice plot over two space periods ($d_l = 2.0$, $b_l = 4.0$, $A = 2.1$, $0 \leq t \leq 100$)

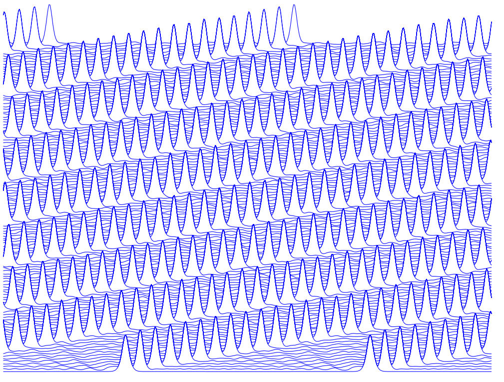

Figure 4. Third solution type: time slice plot over two space periods ($d_l = 2.0$, $b_l = 4.0$, $A = 2.3$, $0 \leq t \leq 100$)

In the subdomain $-0.8 < b_l - 2d_l < 1.6$ one can say that neither the third- nor the fifth order dispersive effects are dominating. In the other words, in this subdomain both dispersive terms (du_{3x} and bu_{5x}) play essential role. In this subdomain the critical amplitude $1.7 \leq A^* \leq 3.41$. Furthermore, the second solution type was detected only in this subdomain for $d_l \leq 1.2$. For $b_l < 2d_l - 0.8$ the fifth order dispersive effects dominate over that of the third order and the value of A^* increases rapidly. For $b_l > 2d_l 1.6$, vice versa, the third order dispersive effects are dominating and the value of the critical amplitude decreases slowly having the limit value $1.67 < A^* < 1.68$.

CONCLUSIONS

In the present paper it is shown that if the amplitude $A > A^*$ then the solitary wave (2) can travel with a constant speed and without significant changes in its amplitude. However, numerical experiments with solitary waves having different amplitudes (and therefore different speeds) demonstrate that they do not interact like solitons — there exist a certain transformation of energy and/or mass between interacting solitons. This phenomenon causes the slower solitary wave move more slowly and the faster one even faster. The results are important for the nondestructive evaluation of material properties of microstructured solids.

References

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