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OPTIMAL DESIGN OF ELASTO-PLASTIC STRUCTURES SUBJECTED TO NORMAL LOADS AND EARTHQUAKE

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Summary The optimal design of elasto-plastic structures subjected to multiparameter (normal) loads and earthquake is presented. It is assumed that under normal loads the structure remains in elastic stage. In case of earthquake, different approximate methods are used in which the plastic reserve of the structure and viscous effects are also taken into account. Introducing bounds for the elastic and permanent deflections a unified optimal design method is elaborated in which both the normal loads and the earthquake are simultaneously taken into consideration. The proposed method is based on the finite element method and on the concept of porous material where the material distribution is described by the densities of the finite elements, which are considered design variables. Numerical test examples are presented.

INTRODUCTION

An important practical application of dynamic plasticity is the investigation of structures subjected to extreme loads as explosion, impact and earthquake [1,2]. In these accidental situations plastic deformations can not be avoided but the structure must be able to absorb the kinetic energy corresponding to the extreme loads without excessive residual displacements and collapse. At the same time, under normal working conditions the structure has to carry the service loads in elastic stage with limited deflections.
The paper presents the optimal layout design of elasto-plastic structures subjected to multiparameter service loads and earthquake. In the solution both the normal and the extreme loads are simultaneously taken into consideration. Considering the normal loads, limits are introduced for the elastic deflections. The investigation of the earthquake is based on approximate methods. The simplest approximation is that the dynamic analysis is reduced to a static analysis where the corresponding loads are determined on the basis of the natural frequency and the stiffness properties of the structure. In case of the pushover method suggested by several Building Codes, the original structure is replaced by a fictitious one-degree-of-freedom elasto-plastic system. Here the properties of this system are calculated also from the natural frequency and from the results of a load history analysis of the original structure and from the diagrams obtained on the basis of the detailed analysis of the El Centro earthquake [3,4].
The mathematical formulation is based on the finite element method and on the concept of porous material where the material distribution is described by the densities of the finite elements, which are the design variables.
The formulation of the problem leads to two nonlinear mathematical programming problems and a natural frequency analysis coupled by the design variables. The solution can be based on iterative procedure.
The application of the proposed method is illustrated by test examples.

PROBLEM FORMULATION

The aim of the proposed approximate method to design a structure which
- -has satisfactory strength to carry the normal loads in elastic stage and the earthquake in plastic stage without collapse,
- -has limited elastic deformations under the action of the normal loads and satisfactory ductility to prevent excessive plastic deformation and fracture during earthquake,
- -has minimum volume.

The standard finite element formulation of the problem is as below. Beside the optimality condition

$$V = \rho_0 h_0 \sum_{i=1}^{n} \Delta_i x_i = \min!$$

in case of the normal loads the equilibrium equation $\mathbf{Q}^{eh} = \mathbf{F}^{-1}(x_i)\mathbf{G}\mathbf{K}^{-1}(x_i)\mathbf{P}_h, \ (h = 1, 2, ..., s);$

the stress limits $-Q_i^{e0} \le Q_i^{eh} \le Q_i^{e0}, \ (h = 1, 2, ..., s), \ (i = 1, 2, ..., n);$

and the constraints for deflections

$$\sum_{i=1}^{n} \frac{Q_i^{eh}}{EI} Q_{ji}^{s} \Delta_i - u_{j0} \le 0;$$

$$\mathbf{G}^{*}\mathbf{Q}_{\mathbf{j}}^{\mathbf{s}} + \mathbf{S}_{\mathbf{j}} = \mathbf{0};$$

$$\left|\mathbf{S}_{\mathbf{j}}\right| = 1, \quad (h = 1, 2, ..., s), \quad (j = 1, 2, ..., t);$$

have to be satisfied.
In case of earthquake first the fundamental natural frequency ω_0 has to be calculated and the optimal plastic design of the structure has to be formulated:

$$\left|\mathbf{K} - \omega^2 \mathbf{M}\right| = \mathbf{0}$$

and

$$V = \rho_0 h_0 \sum_{i=1}^{n} \Delta_i x_i = \min!$$

$$\mathbf{G}^* \mathbf{Q}^\mathbf{p} + \mathbf{R} = \mathbf{0}\,,$$

$$-Q_i^{0p} \le Q_i^p \le Q_i^{0p}, \quad (i = 1, 2, ..., n);$$

Then following the guidelines of the codes and the pushover method presented in [4] in terms of ω_0 and the results of an elasto-plastic load history analysis the fictitious one-degree –of-freedom system can be constructed which provides the target displacement d_t of the original structure. Introducing an allowable displacement d_{ta} the behaviour constraint is expressed as :

$$d_t - d_{ta} \le 0.$$

In the above equations the following notations are used: $\mathbf{P}_h$, (h=1,2,...,s) multiparameter loading, $\mathbf{Q}_\mathbf{h}^\mathbf{e}$, $\mathbf{Q}^\mathbf{p}$ internal forces in elastic and plastic stage, Q_j^s internal forces due to the dummy forces S_j acting at the points (j=1,2,…,t), **R** external loads corresponding to earthquake, Q_i^{e0} and Q_i^{p0} elastic and plastic limit moments of the elements (i=1,2,…,n), $x_i = \frac{\rho_i}{\rho_0}$ dimensionless variables, ρ_0 density of the material, Δ_i length of the finite elements, h_0 a constant, $\mathbf{G}^*$, $\mathbf{F}$, **K, M** equilibrium, flexibility, stiffness and mass matrices.

The unified treatment of the two loading cases is defined by two nonlinear mathematical programming problems and a natural frequency analysis coupled by the design variables x_i. For the solution, an iterative procedure or multicriteria optimization method [1,2] can be used. The proposed method can be applied to the approximate optimal design of frames and wall structures. The application is illustrated by examples. The method has been already extended to other structural problems [1,2].

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