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A NEW FINITE ELEMENT FORMULATION BASED ON THE THEORY OF A COSSERAT POINT

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Summary A finite element formulation based on the theory of Cosserat points is presented. First attempts reveal, that it is free of showing undesired hourglassing or locking-phenoma and additionally performs well for incompressible materials, large deformations and sensitive geometries. Results of tests of this formulation, that has been developed by [3], are presented, as well as first attempts on how to overcome difficulties with deformed initial meshes.

FE-FORMULATION BASED ON THE THEORY OF A COSSERAT POINT

The theory of Cosserat points is the basis of a finite element formulation, that recently was presented by [3]. Within the theory of Cosserat points, the position vectors $\mathbf{X}$, $\mathbf{x}$ can be described through director vectors $\mathbf{D}_i$, $\mathbf{d}_i$

$$\mathbf{X} = \sum_{i=0}^{7} N^i \mathbf{D}_i, \qquad \mathbf{x} = \sum_{i=0}^{7} N^i \mathbf{d}_i, \tag{1}$$

where N^i are shape functions. The geometry of an 8-node-hexahedral element can henceforth be expressed in terms of eight director vectors in the initial and the current configuration respectively. As opposed to standard shape functions, the functions N^i are here defined as

$$N^0 = 1,\ N^1 = \theta^1,\ N^2 = \theta^2,\ N^3 = \theta^3,\ N^4 = \theta^1\theta^2,\ N^5 = \theta^1\theta^3,\ N^6 = \theta^2\theta^3,\ N^7 = \theta^1\theta^2\theta^3, \tag{2}$$

with θ^i being the coordinates in the reference configuration. The approximation is trilinear as for standard 8-node-hexahedrons, but the resorting of the shape functions enables to split the deformation into homogeneous and inhomogeneous parts. The homogeneous part of the deformation gradient **F** is connected to the first three director vectors

$$\mathbf{F} = \sum_{i=1}^{3} \mathbf{d}_i \otimes \mathbf{D}^i. \tag{3}$$

This split requires to find expressions for the material stiffnesses for the homogeneous part of the deformation as well as for each of the inhomogeneous deformation modes. But once this is done, the deformation is decoupled and locking as well as hourglassing-phenomena do not occur. The Cosserat point element does henceforth not require artificial stabilization parameters. Currently, the calculation of the deformation mode related stiffnesses is based on analytical solutions for each of the deformation modes. These analytical solutions however assume a parallelepiped-shaped reference element, with a geometry similar to the initial geometry.

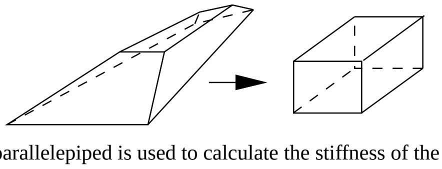

Figure 1: A similar parallelepiped is used to calculate the stiffness of the deformation mode.

This method shows similarities to a class of finite elements using an hour-glass controlled stabilization method, that requires the calculation of artificial stabilization parameters, see e. g. [1], [2], [5] and [4]. Furthermore, stabilized elements require to invert matrices on the element level, which is not necessary for the Cosserat point element.

TESTING THE COSSERAT POINT ELEMENT

The Cosserat point element has been tested concerning a number of different aspects.

- thin structures (beams, shells, plates)
- incompressibility
- distorted elements

These tests have shown, that the performance of the Cosserat point element is excellent for regular shaped elements. It does not show hourglass or locking phenomena but excellent behaviour in case of incompressible materials, for large deformations and in case of sensitive geometries such as shells or plates. One of the tests, which is illustrated in the figure below, is significantly representative for the properties of the Cosserat point element. A cantilever beam is loaded in bending. The beam is discretized with two elements, which are increasingly distorted (Parameter a).

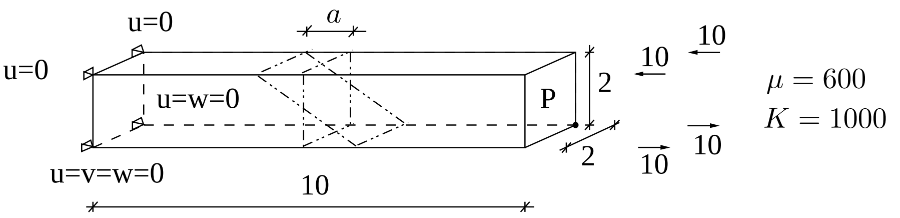

Figure 2: Model of cantilever beam

	Q1	Q1SP	Cosserat	exact
a=0.0	0.28	1.00	1.00	1.00
a=3.0	0.08	0.64	0.16	1.00
a=5.0	0.03	0.03	0.03	1.00

The Cosserat point element gives the exact solution for parallelepiped-shaped elements ($a = 0$), but increasingly wrong results, the more the geometry of the initial mesh differs from a parallelepiped shape. The standard displacement Q1-element locks independently of the shape of the elements.

The difficulty with irregular initial element geometries mainly occurs due to the determination of stiffnesses based on parallelepipeds that are similar to the real element geometry, but might still have an entirely different behaviour in bending or shear. Methods to overcome this difficulty can be based on the numerical analysis of the stiffenss of an irregular shaped element, as well as analytical solutions if those exists.

References

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