ASYMPTOTIC STUDY OF IMPERFECT INTERFACIAL BONDING IN PERIODIC COMPOSITE MATERIALS

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Summary An asymptotic approach for analytical study of the mechanical behaviour of composite materials is proposed. We start with the asymptotic homogenization method. The cell problem is solved by means of a boundary perturbation technique. In order to simulate the phenomenon of the imperfect interfacial bonding we introduce an artificial layer between the components and tend its thickness to zero. As the results effective moduli and local stresses are evaluated for all values of the components' volume fractions and properties.

INTRODUCTION

Effects of imperfect bonding between components play a crucial role in the functionality and reliability of composite materials. It is evident that partially debonded composites are characterized by weaker effective moduli than perfectly bonded ones. Moreover, low strength of the interface between inclusions and a matrix can also significantly change the behaviour of the structure on micro level. In most cases a weak interface response leads to the development of dislocations and voids, which dramatically decreases the strength of the whole material.

Simulation of the imperfect bonding attract considerable attention of many authors. Important results are obtained for prediction of the debonding evolution from initial decohesion through complete separation with a subsequent voids' growth. For this aim Needleman [1] has proposed a specific potential function governing the mechanical response of the interface depending on the displacements' jump across it. Needleman's model has received further development in number of papers of other authors. This interface's description is a phenomenological one, and for its practical implementation it is necessary to know a number of input parameters: the maximal interfacial traction, characteristic lengths of the interfacial displacements and the interface's shear-to-normal strength ratio, which are expected to be determined in experimental way.

In the present paper a new asymptotic approach for simulation of the imperfect interfacial bonding is proposed. We introduce between the components an artificial intermediate layer and tend its thickness to zero. In the asymptotic limit varying the elastic properties of this layer we are able to simulate different types of the interface response. The basic idea of our interface model is in certain sense similar to the approaches proposed by Benveniste & Miloh [2] and Hashin [3], who also considered a thin coating layer between the components for a description of imperfect bonding. However, in contrast to the previous investigations, an advantage of our work is the possibility to develop analytical solutions valid for all degrees of the debonding and for all values of the components' volume fraction and stiffness.

COMPOSITE MATERIALS UNDER CONSIDERATION

As an illustrative example we consider a longitudinal shear deformation of composites consisting of isotropic matrixes and regular square and hexagonal arrays of circular fibres. This stress-strain state may correspond to torsion of the fibre-reinforced bar about the longitudinal axis. In order to simulate the phenomena of imperfect interfacial bonding an artificial intermediate layer of a volume fraction c_0 and a non-dimensional rigidity λ_0 is introduced between the components. We derive a solution for such three-component structure and finally tend $c_0, \lambda_0 \to 0$. Then in the asymptotic limit the intermediate layer disappears and depending on the ratio λ_0/c_0 we can simulate different types of the interface response and describe different rates of debonding. The non-dimensional debonding parameter is $\alpha = (1+\lambda_0/c_0)^{-1}$. Here $\alpha = 0$ corresponds to the perfect interfacial bonding, $\alpha = 1$ corresponds to the complete separation of the matrix and inclusions, any value of α between zero and one represents imperfect bonding. From the mechanical point of view the proposed asymptotic model corresponds to the equality of stresses across the interface (the equilibrium condition of the medium is satisfied) and to a jump of displacements (the continuity condition is not satisfied so that a certain sliding occurs).

THE ASYMPTOTIC PROCEDURE

We start with the asymptotic homogenization method [4, 5], which allows to separate *slow* and *fast* components of the solution. *Slow* components describe the behaviour of the composite on macro level, within the whole sample of the material. *Fast* ones represent local oscillations of stresses and displacements on the scale of heterogeneities and correspond to the micromechanical response of the inhomogeneous medium. *Fast* components should be determined from a so called cell boundary value problem, which is considered within one distinguished periodically repeatable unit cell of the composite structure. We derive an approximate analytical solution of the cell problem by means of the boundary perturbation method [6]. Knowing the local stresses we can predict the appearance of plastic domains and evaluate the effective elastic moduli.

The developed analytical solutions are valid for all values of the inclusions' volume fraction and properties. In particular, they work well in cases when rapid oscillations of stresses occur on micro level (e.g., in the case of perfectly rigid nearly touching fibres, in the case of inclusions with considerably thin coating etc.), when FEM simulation may face principal difficulties.

NUMERICAL RESULTS

At Fig. 1 we present numerical results for the effective longitudinal shear modulus $\langle G \rangle$ (normalized to the shear modulus of the matrix G_1) for the square array of circular fibres (fibres' volume fraction c_2 varies from 0 to $\pi/4$). Data for different values of the debonding parameter α are displayed. In the case of perfect bonding ($\alpha = 0$) the present solution (solid curves) is compared with the results of Perrins et al. [7] (boxes). In the case of complete debonding ($\alpha = 1$) inclusions do not take part in the longitudinal shear deformation, so the solution shows a good agreement with the results for fibrous holes ($G_2/G_1 = 0$, G_2 is the shear modulus of the fibres) [7].

The distribution of the interfacial stress τ_0 (normalized to the homogenized stress $\langle \tau \rangle$) with respect to the polar angle θ is shown in Fig. 2. The data for only a quarter of the unit cell ($\theta = -\pi/4 \ldots \pi/4$) are displayed, for other values of θ the interfacial stress τ_0 continues periodically. It can be easily seen that for a perfectly bonded composite with rigid fibres ($G_2/G_1 > 1$) τ_0 reaches maximum in the points $\theta = \pi n/2$, $n = 0, 1, 2 \ldots$, where the distance between neighbouring fibres is minimal (i.e., interaction between the fibres is maximal). These results allow us to predict the appearance of plastic areas and to calculate the effective initial yield limit, which corresponds to the beginning of the plastification in the composite medium.

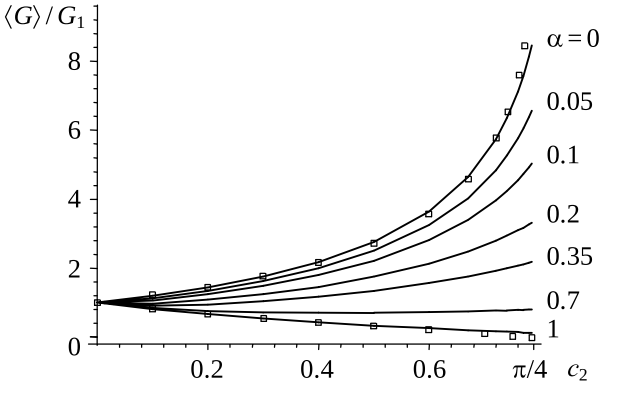

Fig. 1. Effective shear modulus $\langle G \rangle$; $G_2/G_1 = 20$.

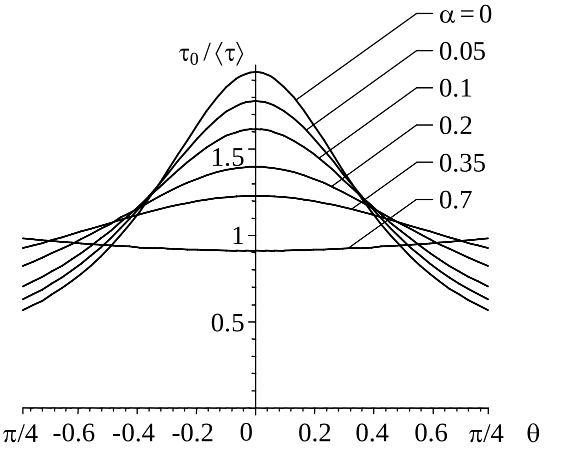

Fig. 2. Interfacial stresses τ_0; $G_2/G_1 = 20$, $c_2 = 0.72$.

CONCLUSIONS

An asymptotic approach for analytical study of the mechanical behaviour of composite materials is proposed. We start with the asymptotic homogenization method. The cell problem is solved by means of a boundary perturbation technique. In order to simulate the phenomenon of the imperfect interfacial bonding we introduce an artificial layer between the components and tend its thickness to zero. As the results effective moduli and local stresses are evaluated in the whole range of the components' volume fractions and properties. The developed approach may be extended to study of different stress-strain states of various periodically inhomogeneous media.

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