INTERFACE RECONSTRUCTION IN CYLINDRICAL TWO-COMPARTMENT-SYSTEMS USING MAGNETIC FIELD TOMOGRAPHY

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Summary In magnetic fluid dynamics appears the problem of reconstruction of free boundary between conducting fluids. The reconstruction problem of the interface between two conducting fluids with different conductivities using external magnetic field measurements in the case of a highly simplified model of an aluminium electrolysis cell has been investigated. In the paper, an interface reconstruction technique based on genetic algorithms is presented, and numerical simulations are compared with some magnetic field measurements.

INTRODUCTION

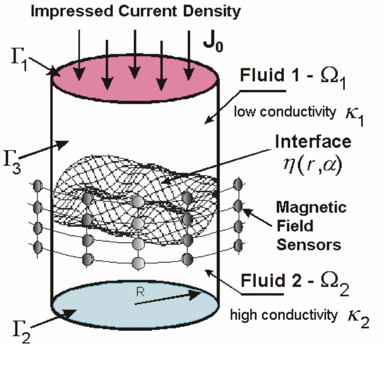
There are a variety of problems in material processing where it would be useful to know the time-dependent distribution of the electrical conductivity of a single fluid or a multiphase flow. For instance, the knowledge of the position of the interface between highly conducting molten aluminium and poorly conducting electrolite (cryolite) is important to prevent unwelcome instabilities in aluminium reduction cells [1]. Recently, it has been demonstrated that the concept of *Magnetic Field Tomography (MFT)* can be successfully used for detection of interfaces between current carrying fluids of different electrical conductivity [2]. We have demonstrated that the external magnetic field generated by the electrical current flowing in a highly simplified model of an aluminium reduction cell provides sufficient information to reconstruct the unknown interface characteristics. In the reconstruction process we have applied genetic algorithms. Genetic algorithms (GAs) – search techniques based on the mechanism of natural evolution and genetics – are particularly effective when the goal is to find an approximate global minimum (or maximum) in a multimodal function domain. In the present work, we show how this technique can be applied to *magnetic fluid dynamics (MFD)*. Therefore, in this paper numerical simulations of the identification of the interface in a cylindrical two-compartment-system are compared with some magnetic field measurements taken from a simple test model.

MAGNETIC FIELD MODELING

If we consider typical figures of aluminum electrolysis cells it must be noticed that the cross section has a length of about L = 10 m, whereas the interface displacement η is very small compared to the lateral extent of the system. Industrial practice shows that already such small interface displacement can perturb significantly the operation of the cell [4]. Consequently, our physical model is characterized by a very small ratio η/L. The considered problem is shown in Fig. 1. Two fluids with different electrical conductivities σ_1 (*top*) and σ_2 (*bottom*) are situated in a long cylinder with the radius R. The cylinder walls are non-conducting. Along the length axis of the cylinder a homogeneous electrical current density $\mathbf{J_0}$ is applied. Then the complete interface perturbation $\eta(r,\alpha)$ can be described [2] as

$$\eta(r,\alpha) = A \sum_{m=-M}^{M} \sum_{n=1}^{N} \eta_{mn} J_m(k_{mn} r) e^{jm\alpha}$$

The value n is called the *radial* mode number and the value m the *azimuthal* mode number. Although the quantity of modes is usually unlimited, the highest modes have the smallest amplitudes and can be neglected. The abbreviation η_{mn} is used for the description of the interface shape. The validity of the above interface representation is limited by the amplitude of the interface deformation. We consider only small deformations because the larger oscillations lead to instabilities due to drop formation.



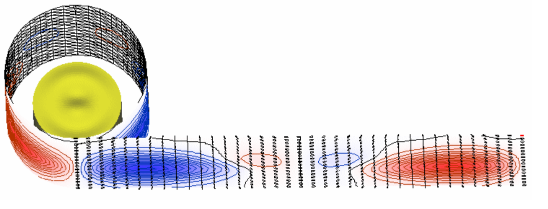


Fig.1: Highly simplified aluminium electrolysis cell model with a non-axisymmetric interface (*left*). Computed sample distribution of the magnetic flux density (B_r component) around cylinder and on the enveloped cylindrical surface (*right*).

If the interface between fluids is flat, the current density **J** is homogeneous everywhere in the cylinder. As soon as the interface deviates from its flat shape due to interfacial waves or an external forcing, the current density **J** will become inhomogeneous near the interface. The inhomogeneity of **J** can be represented by the perturbation current density **j**. If the perturbation of the fluid interface is non-axisymmetric, it leads to a perturbation of the *r*- and *z*-components of the magnetic field outside the cylinder. To model the magnetic field we have to calculate first the current density distribution in the cylinder. This can be done by applying the finite element method. With the distribution of the current density **j** in the cylinder volume, the magnetic flux density around the cylinder is calculated using the Biot-Savart law.

THE INVERSE PROBLEM

The inverse problem is formulated as follows: having the magnetic field flux density distribution (B_r, B_z components) in the sensors positions around the cylinder we would like to reconstruct the interface shape described by (1). The flowchart of the procedure for the interface reconstruction is shown in Fig. 2.
In the current state it enables to identify the modes of the interface between two conducting fluids on the basis of simulated data. It consists of two basic parts: simulation and reconstruction. In the first part for the chosen mode, e.g. mode η_{13}, the magnetic flux density distribution at the sensors positions is calculated using FEM.
In the second step the modes of the interface is identified by means of a modified GA. The goal of the modification is to overcome the problem of the premature convergence in the case of the simple GA [3]. We have applied a multiple optimization loop accompanied by a scaling block which enables to rescale the initial vector of each simple GA run. Thus, the starting point for each run is the same but the parameter vectors are scaled to the best solution of the last step.

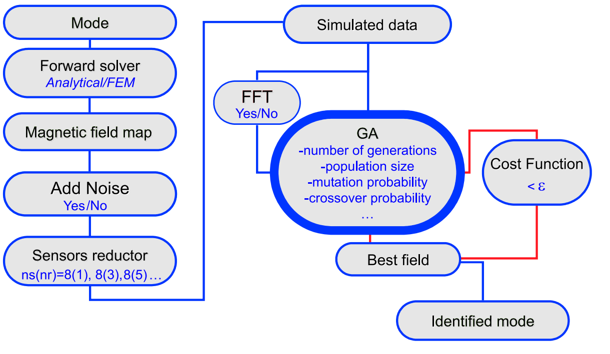


Fig. 2: Flowchart of the mode identification procedure applying a modified GA and FFT

NUMERICAL SIMULATIONS

In our simulations we have assumed that the oscillating interface is in a steady state, i.e. we record the signal from the sensors and then we choose the time instant in which the signal reached the maximum. That time moment corresponds to the maximum of the amplitude of the interface which is identified with the presented procedure.
In the modified reconstruction procedure, we have used a C++ library of genetic algorithm components (GAlib) developed at the M.I.T. [5].
The cost function has been defined as

$$CF = \sqrt{\sum_{i=1}^{Ns} \left(B_{ri} - B_{ro}\right)^2 + \left(B_{zi} - B_{zo}\right)^2},$$

where *Ns* denotes the number of sensors, and B_i, B_o are the values of the magnetic flux density for the current generation and the simulated input data, respectively. They were evaluated using a FEM code.
In the presentation the results of the numerical simulations will be given in full details. The comparison with first measurements shows a good agreement, the existing deviations will be discussed.

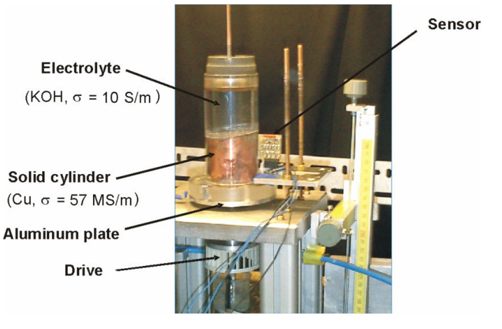


Fig.3: Cylindrical test model of a 2-compartment-system; for the simulation of multi-channel measurements the cylinder was rotating with very low frequency (4-5 Hz).

CONCLUSIONS

We have shown that it is possible to reconstruct the shape of the interface between two conducting fluids on the basis of magnetic flux density simulations with a good quality using genetic algorithms. The new implementation of a modified GA version in the reconstruction procedure improves the quality of reconstruction. Further tests of the algorithm have to be performed to assure proper reconstruction results also in the case of measured data.

References

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