EXTENDED DESCRIPTION OF PORE SPACE STRUCTURE AND FLUID FLOW THROUGH ANISOTROPIC POROUS MATERIALS

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Summary The work concerns modelling of mechanical behavior of fluid in porous materials with anisotropic pore space structure. A new macroscopic description is proposed in which the motion of fluid in anisotropic pore space of skeleton is considered as a motion of material continuum in Minkowski (anisotropic) metric space. The generalized equations for wave propagation and for Brinkman and Darcy equations describing fluid flow in porous materials have been derived.

BASIC CONCEPTS

In the paper the fluid motion in a rigid porous medium of anisotropic pore space structure is described. Considerations are based on the new macroscopic model of saturated porous medium in which a fluid flow through porous skeleton of anisotropic pore space structure is described as a motion of the material continuum in the plane anisotropic metric space ([1], [2]). This model takes into account the fundamental fact of kinematics of fluid-saturated porous solid that pore space of permeable skeleton forms the real space for a fluid motion and its structure imposes constraints on that motion.
In such approach the anisotropic pore space of permeable materials is modelled as the affine Minkowski space ([3]-[5]) immersed in the Euclidean space beeing the model of the physical space. The distance u between any two points of the pore space is given by the metric

$$u^2 = \mathbf{u} \cdot \mathbf{M}_A(\mathbf{n}) \cdot \mathbf{u}$$

where $\mathbf{M}_A(\mathbf{n}) \in V^* \otimes V^*$ is the metric tensor of Minkowski space that, in general, depends on the direction $\mathbf{n} = \mathbf{u} / u$ of the measured vector $\mathbf{u} \in V$ and also on its sense $(\mathbf{M}_A(-\mathbf{n}) \neq \mathbf{M}_A(\mathbf{n}))$. The dot in the above relation denotes the bilinear operation (dual multiplication) defined on the dual vector spaces V and V^*.
Tensor $\mathbf{M}_A(\mathbf{n})$ characterizes the anisotropic structure of the skeleton pore space. It determines the measures of any line, surface and volume elements in the pore space and together with the metric tensor $\mathbf{M}$ of the Euclidean space may be applied to define the geometrical parameters of pore structure of porous materials: the tortuosity, the surface and volume porosity.
The concept of plane anisotropic space used as a macroscopic model of the skeleton pore space in which a fluid motion takes place exceeds the framework of fundamental concepts of the mixture theory that is the basis for most theories of multiphase media. Its application to the modelling of porous materials is the new proposition of solution of this problem.
The purpose of this paper is to formulate the balance equations and constitutive relations for fluid filling anisotropic pore space of permeable materials and to discuss the influence of pore structure on mechanical behavior of fluid.

KINEMATICS, BALANCE AND CONSTITUTIVE EQUATIONS

To describe the kinematics of fluid motion in the anisotropic pore space, the existence of continuous fields for fluid mass distribution and mass flux was assumed. It enabled one to define the phase and partial mass densities of fluid and its velocity. Similar requirement formulated for the surface forces as balance quantities results in Cauchy theorem relating the phase stress vector $\mathbf{t} \in V$ and the stress tensor $\mathbf{T} \in V \otimes V$ in a fluid filling anisotropic pore space,

$$\mathbf{t}(\mathbf{x}, \mathbf{t}) = \mathbf{T}(\mathbf{x}, \mathbf{t}) \cdot \mathbf{M}_A(\mathbf{N}) \cdot \mathbf{N} \quad ,$$

where $\mathbf{N} \in V$ is the Minkowski unit vector $(\mathbf{N} \cdot \mathbf{M}_A(\mathbf{N}) \cdot \mathbf{N} = \mathbf{1})$ perpendicular to the surface element.
It was shown that the global and local balance equations for mass and linear momentum written in the vector notation take the classical form and are independent of the pore structure parameters. Such description is a direct consequence of independence of the obtained general expressions for fluxes and for the Gauss-Ostrogradski theorem of any space metric.
The constitutive relations for the phase stress in fluid and for its interaction force with porous skeleton have been formulated. It was assumed that fluid flowing through anisotropic pore space is of the Stockes type and to obtain general expression for the interaction force, the additional tensor $\mathbf{A}(\mathbf{v}) \in V \otimes V^*$ was introduced. This tensor characterizes directional distribution of effective number of pores composing their unit surface in a crossection perpendicular to the given direction.

RESULTS

After reduction the system of equations takes form

$$\frac{\mathrm{D}\rho}{\mathrm{D}t} + \rho\,\nabla\cdot\mathbf{v} = 0 \;,$$

$$\mathit{M}(\mathbf{v})\cdot\frac{\mathrm{D}\mathbf{v}}{\mathrm{D}t} = \overline{\nabla}\left\{-\mathrm{p}(\rho)+\mu'\,\nabla\cdot\mathbf{v}\right\} + \mu\,\nabla\cdot\mathbf{M}^{-1}\cdot\nabla\{\mathbf{v}\} - \mathit{R}(\mathbf{v})\cdot\mathbf{v} + \mathit{M}(\mathbf{v})\cdot\mathbf{b}$$

where $\overline{\nabla} = \mathbf{M}^{-1}\cdot\nabla$ and $\mathbf{v}$, ρ, p, $\mathbf{b}$ stand for velocity, mass density, pressure and external mass forces in fluid, respectively. The quantity

$$\mathit{R}(\mathbf{v}) = \mathit{M}(\mathbf{v})\cdot\left(\alpha_1\,\mathbf{I} + \alpha_2\,\mathbf{A}(\mathbf{v}) + \alpha_3\,\mathbf{A}^2(\mathbf{v})\right)$$

is the tensorial coefficient of the fluid flow resistance in porous material with anisotropic pore space structure, and

$$\mathit{M}(\mathbf{v}) = \rho\,\mathbf{M}^{-1}\cdot\mathbf{M}_{\mathrm{A}}(\mathbf{v})$$

can be interpreted as the tensor of fluid mass density in anisotropic space. It is seen that in such description the coefficient $\mathit{R}(\mathbf{v})$ is the function of two, in general, independent tensorial characteristics of the pore structure.
The above system for stationary flow of incompressible fluid reduces to generalized form of Brinkman equation,

$$\overline{\nabla}\{\mathbf{p}\} - \mathit{M}(\mathbf{v})\cdot\mathbf{b} = -\,\mathit{R}(\mathbf{v})\cdot\mathbf{v} + \mu\,\nabla\cdot\mathbf{M}^{-1}\cdot\nabla\{\mathbf{v}\} \;,$$

or Darcy equation,

$$\overline{\nabla}\{\mathbf{p}\} - \mathit{M}(\mathbf{v})\cdot\mathbf{b} = -\,\mathit{R}(\mathbf{v})\cdot\mathbf{v} \;,$$

and for disturbances of small amplitude it takes form of wave equation explicitly dependent on the pore structure parameters

$$\mathit{M}_{\mathrm{o}}(\mathbf{v})\cdot\frac{\partial^2\mathbf{v}}{\partial\mathbf{t}^2} = -a_{\mathrm{o}}^2\,\overline{\nabla}(\nabla\cdot\mathbf{v}) - \mathit{R}_{\mathrm{o}}(\mathbf{v})\cdot\mathbf{v}$$

where a_{o} is the velocity of wave propagation in the bulk fluid, and

$$\mathit{R}_{\mathrm{o}}(\mathbf{v}) = \mathit{M}_{\mathrm{o}}(\mathbf{v})\cdot\left(\alpha_1\,\mathbf{I} + \alpha_2\,\mathbf{A}(\mathbf{v})\right)\;, \qquad \mathit{M}_{\mathrm{o}}(\mathbf{v}) = \rho_{\mathrm{o}}\,\mathbf{M}^{-1}\cdot\mathbf{M}_{\mathrm{A}}(\mathbf{v}) \;.$$

The proposed description allows one to formulate and analyze various problems of fluid flow and wave propagation in porous materials with extended characteristics of pore structure anisotropy and also in materials with nonsymmetric dynamical and filtration properties.

References

[1] M. Cieszko, Minkowski Space as a Model of Anisotropic Pore Space of Permeable Materials. Modelling of the Skeleton Pore Structure. In Proceedings of the International Symposium on *"Trends in Continuum Physics"*, World Scientific, 87-98, 1998.
[2] M. Cieszko, Application of Minkowski Space to Description of Anisotropic Pore Space Structure in Porous Materials, *ZAMM*, **80** S1, 129-132, 2000.
[3] H. Rund, The Differential Geometry of Finsler Spaces, Springer-Verlag, 1959.
[4] A.C. Thompson, Minkowski Geometry, Cambridge University Press, 1996.
[5] D. Bao, S.S. Chern, Z. Shen (2000), An Introduction to Riemann-Finsler Geometry, Springer-Verlag.