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CONTACTLESS INDUCTIVE FLOW TOMOGRAPHY: THEORY AND EXPERIMENT

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Summary When a moving electrically conducting fluid is exposed to an applied magnetic field, electrical currents are induced that give rise to an additional magnetic field. The ratio of the induced field to the applied field is determined by the magnetic Reynolds number Rm. If Rm is not too small, the induced field can be measured in the exteriour of the fluid. Applying the imposed magnetic fields in different directions and measuring the respective induced fields one can gather sufficient information to reconstruct, at least approximatively, the velocity structure of the fluid. The theory of such a *contactless inductive flow tomography* (CIFT) is delineated, and its practical feasibility is demonstrated in a liquid metal experiment.

MOTIVATION

In many industrial applications, including crystal growth and metallurgy, there is a growing interest in determining the velocity field of metal or semiconductor melt flows. Due to the opaqueness of those fluids, the customary optical methods of flow measurements fail. What is more, the fluids are often very hot or chemically aggressive, hence a contactless measuring technique would be highly desirable. In case of electrically conducting fluids, the velocity dependent magnetic induction can be utilized to establish such a contactless measuring technique.

THEORY

When an electrically conducting fluid, moving with the velocity $\mathbf{v}$, is exposed to a magnetic field $\mathbf{B}$, an electromotive force $\mathbf{u} \times \mathbf{B}$ is induced that drives a current $\mathbf{j} = \sigma(\mathbf{E} + \mathbf{v} \times \mathbf{B})$, with σ denoting the electrical conductivity of the fluid and $\mathbf{E}$ the induced electric field. In the steady case, $\mathbf{E}$ can be expressed as the gradient of the electric potential, $\mathbf{E} = -\nabla\varphi$. For this form of the current, the application of Biot-Savart's law yields the following expression for the induced magnetic field $\mathbf{b}$ [1]:

$$\mathbf{b}(\mathbf{r}) = \frac{\mu_0\sigma}{4\pi} \int\limits_V \frac{(\mathbf{v}(\mathbf{r}') \times \mathbf{B}(\mathbf{r}')) \times (\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3} dV' - \frac{\mu_0\sigma}{4\pi} \int\limits_S \frac{\varphi(\mathbf{s}')\mathbf{n}(\mathbf{s}') \times (\mathbf{r} - \mathbf{s}')}{|\mathbf{r} - \mathbf{s}'|^3} dS' \ . \tag{1}$$

The first term on the r.h.s of Eq. (1) represents the effect of the so-called *impressed* currents, $\sigma\mathbf{v} \times \mathbf{B}$, in the fluid volume V. The second term appears if we convert the volume integral over the $\nabla\varphi$ term into a boundary integral. These so-called *secondary* currents depend on the electric potential φ at the fluid boundary S. The electric potential φ, in turn, has to fulfil the boundary integral equation

$$\varphi(\mathbf{s}) = \frac{1}{2\pi} \int\limits_V \frac{(\mathbf{v}(\mathbf{r}') \times \mathbf{B}(\mathbf{r}')) \cdot (\mathbf{s} - \mathbf{r}')}{|\mathbf{s} - \mathbf{r}'|^3} dV' - \frac{1}{2\pi} \int\limits_S \frac{\varphi(\mathbf{s}')\mathbf{n}(\mathbf{s}') \cdot (\mathbf{s} - \mathbf{s}')}{|\mathbf{s} - \mathbf{s}'|^3} dS' \ , \tag{2}$$

which results from Green's theorem when applied to the solution of the Poisson equation for the case of insulating boundaries. Interestingly, Eqs. (1) and (2) are equivalent to the corresponding equations in magneto- and electroencephalography [2], where only the term $\sigma\mathbf{v} \times \mathbf{B}$ has to be replaced by the impressed currents due to the neuronal activity in the brain.

It is important to note that for a small magnetic Reynolds number Rm, defined as $Rm = \mu_0\sigma l v$ (where l and v denote characteristic length and velocity scales of the fluid), the total $\mathbf{B}$ under the integrals in Eqs. (1) and (2) can be replaced by the applied magnetic field $\mathbf{B_0}$. Under this condition, we get a *linearized* version of the general inverse problem of how to infer the velocity field from measuring the induced magnetic field in the exterior of the fluid and the electric potential at the boundary of the fluid [1]. The considerations in [3] had revealed an intrinsic non-uniqueness of this inverse problem with respect to the depth dependence of the velocity field. This non-uniqueness can only be resolved by applying magnetic fields of varying frequencies, or by the use of appropriate regularization techniques.

In [4] it was shown that the electric potential measurement can be avoided by applying the imposed magnetic field in two, e.g. orthogonal, directions and measuring only the respective induced magnetic field. A necessary ingredient of this method is the implicit treatment of the unknown electric potential at the fluid boundary.

EXPERIMENT

An experiment has been set-up in order to demonstrate the feasibility of the CIFT. The basic problem to be solved prior to any practical application is the sufficiently accurate determination of small induced magnetic fields on the background of much larger imposed magnetic fields. Fig. 1 shows the experimental set-up, both in a schematic view and as a photograph. We use 4.4 litres of the eutectic alloy GaInSn in a cylindrical vessel of 18 cm diameter and approximately the same fill

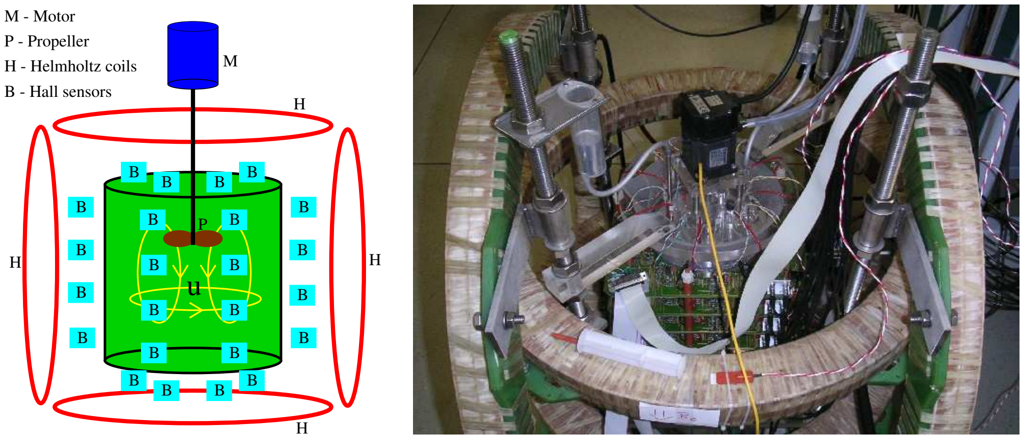

Figure 1. Schematic view (left) and laboratory set-up (right) of the CIFT experiment. The propeller driven flow of a metallic melt is exposed alternately to a transverse and an axial magnetic field, which are produced by two pairs of Helmholtz coils. The respective induced magnetic fields are measured at 49 Hall sensors.

level. A motor driven propeller can reach rotation rates up to 2000 rpm, which corresponds to a magnetic Reynolds number of approximately 0.4.

The measuring procedure of the CIFT is as follows: The current is switched on and off in the two Helmholtz coil pairs, producing alternately a transverse and an axial magnetic field. For either applied field, the respective induced magnetic fields are measured at a total of 49 Hall sensors covering the fluid volume rather homogeneously. Both data sets are put jointly into the inversion solver. After each switching and measuring cycle one gets a global picture of the flow. In the present configuration the flow is monitored every 3-4 seconds.

In Fig. 2 we show the measured induced magnetic fields for transverse (a) and axial (b) field situations, jointly with the reconstructed velocity (c). This measurement was carried out for the case that the propeller pumped upward at a rotation rate of 1200 rpm. Qualitative as well as quantitative changes of the flow field were resolved by the CIFT method in a reasonable and reproducible way. Even the transient start-up of the 2D flows has been resolved in time-steps of about 3 seconds.

Presently, work is under way to include AC fields in order to improve the depth resolution of the method.

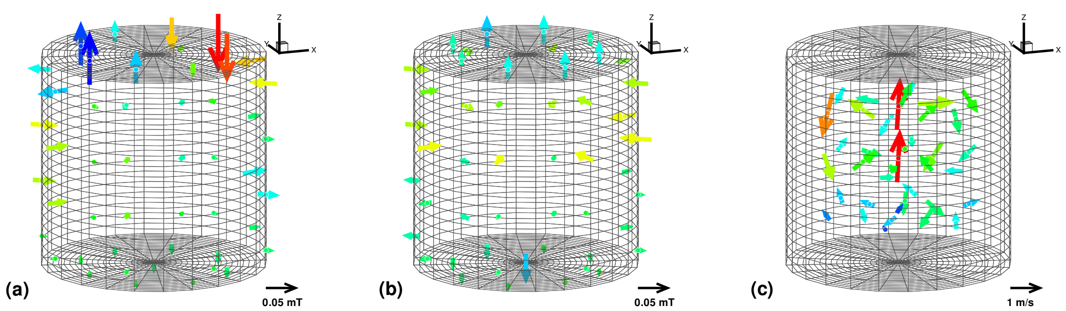

Figure 2. The measured induced magnetic fields for transverse (a) and axial (b) imposed magnetic field. The direction of the arrows indicates the measured field component. The reconstructed velocity field is shown in (c), where the colour indicates the flow speed.

References

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