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MODEL-BASED CONTROL OF SHEAR FLOWS USING LOW-DIMENSIONAL GALERKIN- AND VORTEX MODELS

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Summary The present study proposes a model-based flow control strategy using low-dimensional coherent-structure representations. This strategy is applied to the suppression of a von Kármán vortex street and the mixing enhancement of a wall-bounded shear-layer. Here, low-dimensional Galerkin and vortex models are used as a 'plant' for controller and observer design. The control approach is validated against direct numerical simulations.

INTRODUCTION

Low-dimensionsal coherent-structure models of shear flows play an increasing role in industrial and academic research — in addition to experiment and simulation. Low-dimensional models provide deeper understanding of key processes and enable the examination and design of actuators and the application control theory methods.

A model-based strategy of closed-loop flow control is proposed using a low-dimensional coherent-structure model as a plant for observer and controller design. The models are developed with Galerkin and vortex methods. The first approach typically leads to lower dimensions whereupon the latter ansatz is applicable for a larger dynamic range.

GALERKIN MODEL OF THE ACTUATED FLOW AROUND A CIRCULAR CYLINDER

Firstly, our strategy is applied to the flow around a circular cylinder at Reynolds number 100 [1]. At this Reynolds number the von Kármán vortex street is fully developed (see Fig. 1). The control objective is the supression of vortex shedding in the near wake. Two different actuators are investigated in this study:

A) transversal local volume force in the near wake
B) transversal oscillation of the cylinder

Control design is based on a low-dimensional Galerkin model of the natural flow which is approximated by a Galerkin expansion of the velocity field:

$$\mathbf{u}^{[N]}(\mathbf{x},t) \approx \mathbf{u}_0(\mathbf{x}) + \sum_{i=1}^{N} a_i(t)\ \mathbf{u}_i(\mathbf{x}). \qquad (1)$$

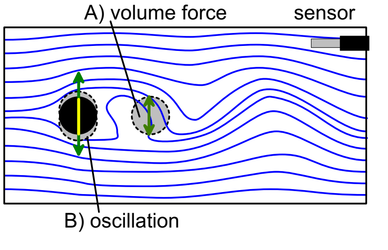

Fig. 1: Streamlines of the von Kármán vortex street in the wake of a circular cylinder at $Re = 100$. Two different actuators are investigated: A) a local volume force, and B) oscillations of the cylinder. The state of the flow is monitored by a single hotwire probe.

The velocity field is decomposed into a basic mode $\mathbf{u}_0$ representing the steady Navier-Stokes solution and N orthonormal modes for the fluctuations. The fluctuations are described by time-dependent Fourier coefficients a_i and spatial POD modes $\mathbf{u}_i$. Low-dimensional Galerkin models usually fail to represent transients, which are essential for control design. To overcome this challenge, a shift-mode $\mathbf{u}_N$ is introduced [2]. This mode is the normalized difference between the steady and mean flow. The shift-mode allows to describe the transient phase. Galerkin projecting (1) onto the Navier-Stokes equation yields the Galerkin system of ordinary differential equations

$$\frac{d}{dt}\, a_i = \frac{1}{Re} \sum_{j=0}^{N} l_{ij}\ a_j + \sum_{j,k=0}^{N} q_{ijk}\ a_j\ a_k\,, \qquad a_0 = 1, \qquad (2)$$

with the system coefficients l_{ij} and q_{ijk}. The dimension of the model in (2) is the number of modes N. A minimal Galerkin model with $N = 3$ captures the dominant leading harmonic of the vortex street and resolves 96% of the turbulent kinetic energy.

In case A) the actuator is modelled by an additional term $g\ \mathbf{b}$ in (2), where $g(t)$ is the controller output and $\mathbf{b}$ is the Galerkin representation of the volume force [3]: $d\mathbf{a}/dt = A(\mathbf{a})\ \mathbf{a} + \mathbf{b}\ g$, with $\mathbf{a} = (a_0;\ldots;a_N)$. An energy-based control is designed for this model. Its purpose is to reduce the energy of the fluctuation modes quantified by $r = \sqrt{a_1^2 + a_2^2}$. The controller reduces the turbulent kinetic energy in the near wake by 45%. In addition, an observer is designed to estimate the state of the flow from a single sensor reading. The results are validated against direct numerical simulation. A major task in the design of the controller is to appreciate the validity range of the model and the natural shedding frequency. In this configuration, model-predicted actuated transients reproduce simulations provided that the Fourier coefficients remain near the invariant manifold of natural transients (see Fig. 2).

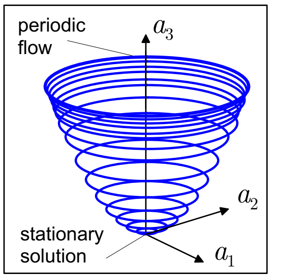

Fig. 2: Invariant manifold of natural transients from the fixed point (steady flow) to the limit cycle (periodic flow).

The oscillating cylinder in case B) is a well-examined actuator. This control generates a transversal cross-flow which is described by an additional actuation mode and its Fourier coefficient a_c. The general form of the Galerkin model for actuation A and B is given by

$$\frac{d}{dt}\ \mathbf{a} = A(\mathbf{a}, g)\ \mathbf{a} + \mathbf{b}\ g + \mathbf{k}_1\ g^2 + \mathbf{k}_2\ \frac{d}{dt}\ g\ . \tag{3}$$

Here, the controller output is equal to the velocity of the cylinder $g = a_c$. The energy-based control method is applied to this model for case B) by using the acceleration of the cylinder instead of the velocity as the controller output ($g = da_c/dt$).

VORTEX MODEL FOR THE FLOW AROUND A BACKWARD-FACING STEP

The 3D flow around a backward-facing step is investigated at a transitional Reynolds number of 4000. The flow separates at the step. In the sequel, the coherent structures are characterized by vortex-shedding and the roll-up of two shear-layer vortices. The flow is controlled by an acoustic actuator located at the upper edge of the step, thus amplifying the Kelvin-Helmholtz-type shear-layer instability. The state is monitored by an array of pressure transducers downstream of the step. The control goal is the regulation of the separation-bubble length. This reattachment length is an indicator of mixing enhancement.

The coherent structures are described by a vortex model [4]. The vorticity is approximated by Oseen vortices in the potential base flow. The dynamics are provided by Biot-Savart's law.

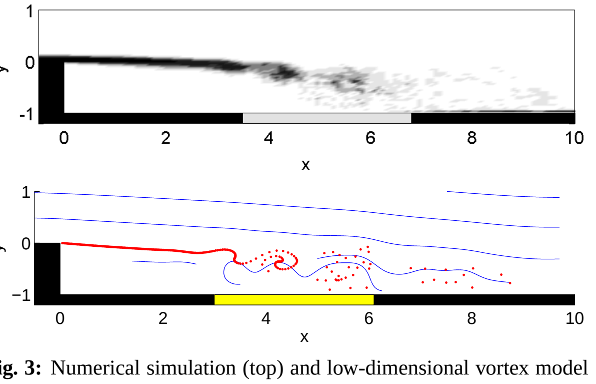

Fig. 3: Numerical simulation (top) and low-dimensional vortex model with $\approx$ 300 vortices (bottom) of the flow around a backward-facing step at $Re_h = 4000$. The top figure illustrates the vorticity distribution of a snapshot. The figure at the bottom indicates vortex locations by red dots. Regions of upstream velocity close to the wall are marked by grey bars.

The manipulation of 2D coherent structures is reproduced with some 300 vortices (see Fig. 3), even for a large dynamic range. As compared to direct numerical simulation, vortex models provide computationally efficient ways for 'rapid prototyping' of coherent structures and the examination of actuation effects. However, the very hybrid nature of this Lagrangian description poses a serious challenge for most control theory methods. This may be overcome by an 'Eulerization' [4].

CONCLUSION AND OUTLOOK

Low-dimensional Galerkin models have been successfully applied to model-based closed-loop control design of a wake instability. Low-dimensional vortex models provide an efficient and ample representation of the coherent structures of shear flows. Hybrid models incorporating the respective strengths of Galerkin and vortex models are pursued as a promising path for more complex configurations.

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