XXI ICTAM, 15-21 August 2004, Warsaw, Poland

APPLICATION OF EXTENDED PHASE SPACE TO INVESTIGATION OF FORCED BIHARMONIC OSCILLATIONS

Kazakevitch Michael I., Volkova Viktorija E.
Dniepropetrovsk National University of Railway Transport named after Academician Lazaryan, 2 Lazaryana str, UA-49010 Dniepropetrovsk, Ukraine

Summary The results of investigation of dynamic behaviour of mechanical systems which oscillations are described by a non-linear Duffing-type equation are presented in the paper. The outer periodic biharmonic excitation is applied to the mechanical system.

The development of the qualitative methods of investigation of dynamic systems, suggested by the authors, is the effective means for identification of dynamic systems. The results of the extensive investigations of the behaviour of linear dynamic systems and symmetrical system with double well potential under polyharmonic excitation are given in the paper. The bases of the method of qualitative investigation of oscillations were developed by Poincaré. Application of these methods is most effective for the investigation of oscillations of systems with one degree of freedom. The classical approach to qualitative investigation of oscillations consists in finding out special points on a phase plane $(y, \dot{y})$ and definition of their type (node, saddle, centre or focus). Studying of special points of system explains the picture of trajectories of points on a phase plane (displacement, velocity) in their neighbourhood, however does not allow to study oscillatory processes finally.
Phase space of dynamic systems is multi-dimensional. Each point of this space is characterized by not less than four co-ordinates. In particular: displacement, velocity, acceleration and time. Real space has three dimensions. It is more convenient for the analysis. We consider the phase space as limited to three dimensions, namely displacement, velocity and acceleration. Another choice of parameters of phase planes is also possible [1, 2]. Phase trajectory on a plane $(y, \ddot{y})$ is of the greatest interest. It is known that accelerations of points are more sensitive to deviations of oscillations from harmonic ones.
It is connected with the fact that power criteria on it are interpreted most evidently. Besides, dependence $\ddot{y}(y)$ is back symmetric relative to axis y of the diagram of elastic characteristic. For example, in Figure 1 diagrams of change of the elastic characteristic and acceleration for the system with “backlash” are shown. Only the phase trajectories $\ddot{y}(y)$ allow establishing a type and a level of non-linearity of a system.

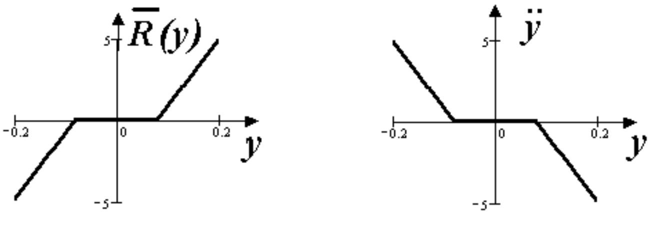

Figure 1. Diagrams of change of elastic properties and acceleration for system with “backlash”.

Differential equation of polyharmonic forced oscillations
Let’s remark that outer excitation can contain some harmonics for wide range of mechanical dynamic systems. Their amplitudes can be various. The forced oscillations of such systems are described by the non-linear differential equation of the type

$$\ddot{y} + \varepsilon \dot{y} + R(y) = F(t);$$
$$F(t) = F_0 + \sum_{i=1}^{n} F_i(t) \cos(\omega_i t) + \sum_{j=1}^{n} F_j(t) \sin(\omega_j t), \; i = 1, 2, 3 \ldots n, \; j = 1, 2, 3 \ldots n, \tag{1}$$

where y is the generalized coordinate; ε is the damping coefficient of the system; $R(y) = -\alpha y + \beta y^3$ is the elastic characteristic of the system; and F_0, F_i, F_j, ω_i are parameters of the outer polyharmonic excitation.
Let’s restrict our investigation to symmetrical biharmonic oscillations, then outer polyharmonic excitation has the following form:

$$F(t) = F_1 \cos(\omega_1 \ t) + F_m \cos(\omega_m \ t), m = 1,2,3\ldots, \quad (2)$$

The excitation is monoharmonic in a case if $m = 1$. The results of investigation for $m = 2, 3$ are presented in the paper. We compare a linear system with a non-linear symmetric system.

The possibility of occurrence of the non-adjacent stable oscillations at the fixed frequency of excitation is the peculiarity of the investigated systems. The realization of one of the stable regimes of oscillations depends on the initial conditions in a complicated manner. The frequencies of "large" oscillations stall for the cases of monoharmonic and biharmnoic excitation are different. It is important that "skeleton" curves for oscillations on fundamental tone, ultra- and sub-harmonic oscillations have different angles. The amplitude of oscillations within the frequency range of main resonanse is larger then if it was a monoharmonic excitation. As shown in Figure 2 a-b, for all the types of "large" oscillations the phase trajectories are back symmetrical relative to axis y of the diagram of elastic characteristic. It allows to recognise the type of dynamic system.

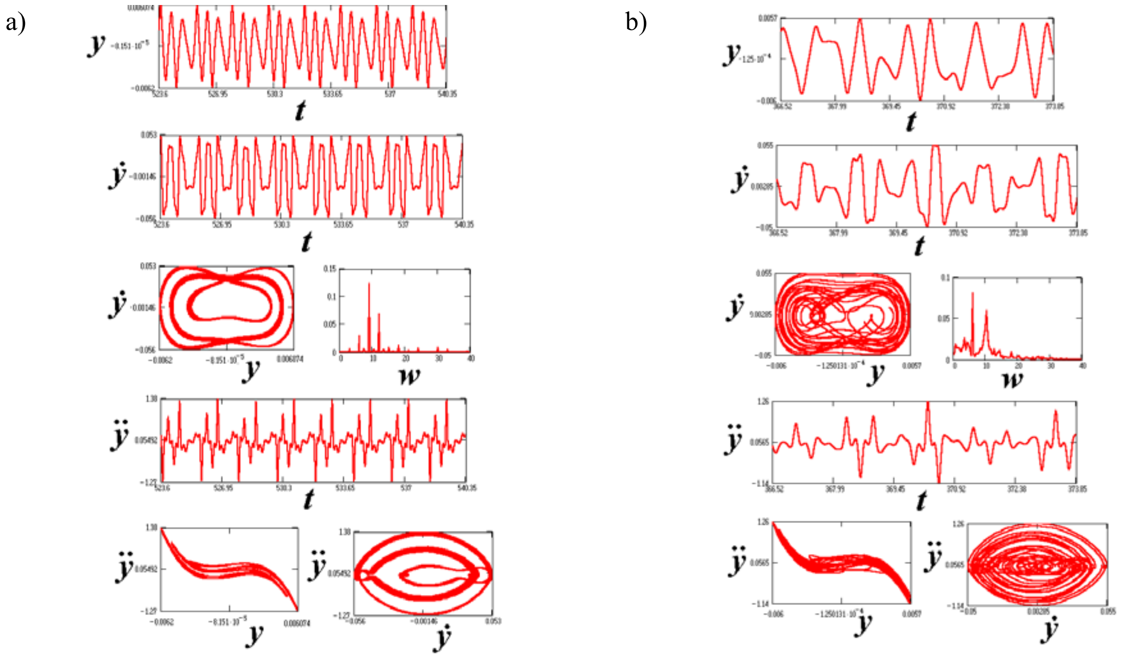

Figure 2. Time characteristics and phase trajectories of the system under consideration ($m = 3$; $\varepsilon = 0.5\ s^{-1}$; $\alpha = 408\ s^{-2}$; $\beta = 7660000\, m^{-2} s^{-2}$; $F_1 = 0.15\ ms^{-2}$; $F_1 = 0.075\ ms^{-2}$): a) combinative oscillations; b) chaotic oscillations.

CONCLUSIONS

The development of qualitative methods of investigation of dynamic systems suggested by the authors is effective means of analysis and identification of dynamic systems. Simultaneous use of all three types of signals registered in time, namely displacement, velocity and acceleration allows to expand considerably the opportunities of traditional methods of investigation. The use of the given phase trajectories enables us to determine with a high degree of reliability the following peculiarities:

- presence or absence of non-linear character of behaviour of a dynamic system;
- type of non-linearity;
- type of dynamic process (oscillations of the basic tone, combinative oscillations, chaotic oscillations.).

Unlike existing asymptotic and stochastic methods of identification of dynamic systems, the use of the suggested technique is not connected with the use of a significant amount of computing procedures, and also it has a number of advantages during investigations of complicated oscillations.

References

[1] Kazakevich, M.I. and Volkova, V.E.: "Dynamics of systems with double well potential". - Dnepropetrovsk: Art-Press. – 2000. - 169 pp. (in Russian)

[2] Kazakevitch, M.I. and Volkova, V.E.: "Identification of non-linear dynamic systems". Proc. 7-th International Conference on Modern Building Materials, Structures and Techniques, vol. 3. - Vilnius: Technika. – 2001. - :P. 145-150.