XXI ICTAM, 15–21 August 2004, Warsaw, Poland

STIRRING BY BLINKING ROTLETS IN A BOUNDED STOKES FLOW

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Summary A blinking rotlet model is used for the analysis of stirring in a Stokes flow in a rectangular domain. After the two-dimensional biharmonic equation is solved analytically, the associated velocity field of a pair of blinking rotlets positioned symmetrically on the y-axis, is used studying the stirring qualities of this blinking rotlet model. Contour kinematic simulations are performed in order to obtain information about the chaotic behaviour of a blob of passive tracer material put in this flow field.

INTRODUCTION

The way in which passive tracer material will be distributed in an ambient fluid depends basically upon two processes: advection due to the presence of an imposed velocity field and secondly upon mass-diffusion. Although mixing is essentially a combination of both mechanisms, only the effects of advection are considered. Strictly speaking, with this limitation one does not deal with mixing anymore, but with stirring.

The spreading of tracer particles through advection becomes more efficient when the flow has a chaotic nature. This chaotic nature is characterized by an exponential growth in time of the contact length of a patch of tracer material with its surroundings. Time-dependency is in 2D an essential ingredient towards 'good stirring' [1] — what is meant by this, not to mention how to quantify it, is not just of importance for a good understanding of stirring, but also of mixing.

In this study the behaviour of tracer material is considered when put in an incompressible, highly viscous fluid. The velocity field is driven by two, independently adjustable rotlets. In order to advect the tracer material chaotically, a blinking rotlet model is used, by which a time-dependent laminar flow is obtained. This blinking rotlet model was studied before by Aref & Meleshko [2]; they considered the stirring effects of two rotlets in a circular domain.

PROBLEM FORMULATION AND SOLUTION APPROACH

A highly viscous, incompressible fluid is captured in a rectangular domain Π: $\Pi = \{\mathbf{x} \in \mathbb{R} | -a < x < a, -b < y < b\}$, and is stirred by two individually adjustable rotlets. The resulting velocity field is described by an overall stream function Ψ; letting the rotlets blink, implies that Ψ is actually a function of time, although not continuously.

The position of the rotlets are restricted such that they are confined along the y-axis: rotlet A is positioned at $(0, c_0)$ and rotlet B at $(0, -c_0)$; $\pm c_0$ are stagnation points which depends on a and b. Thus a blinking rotlet model of two rotlets can be constructed in which the rotlet that is "off", does not disturb the flow. So, one writes

$$\Psi(t, \mathbf{x}) = \varepsilon_A(t)\Psi_A(\mathbf{x}) + \varepsilon_B(t)\Psi_B(\mathbf{x}), \qquad (t, \mathbf{x}) \in \mathbb{R}^+ \times \Pi, \tag{1}$$

where the time-dependent functions $\varepsilon_{A,B}(t)$ are periodic square waves used to adjust the intensity of the rotlets and to switch the rotlets "on" and "off". The time-independent contributions Ψ_A and Ψ_B satisfy the biharmonic equation and can be decomposed into two, what might be called, elemental stream functions $\Psi_1(\mathbf{x})$ and $\Psi_2(\mathbf{x})$, in such a way that

$$\Psi_A(\mathbf{x}) = [\Psi_1(\mathbf{x}) + \Psi_2(\mathbf{x})], \tag{2}$$

$$\Psi_B(\mathbf{x}) = [\Psi_1(\mathbf{x}) - \Psi_2(\mathbf{x})]. \tag{3}$$

Due to the linearity of the biharmonic equation, both Ψ_1 and Ψ_2 have to be solutions of the biharmonic equation. The main objective therefore is to find the analytical expressions for these elemental functions satisfying the appropriate, with no-slip associated, boundary conditions.

The underlying idea solving the biharmonic equation is that the elemental stream functions Ψ_i $(i = 1, 2)$ are written as a sum of two contributions

$$\Psi_i(\mathbf{x}) = \psi_i^{(\sigma)}(\mathbf{x}) + \psi_i(\mathbf{x}), \quad i = 1, 2. \tag{4}$$

The first contribution takes the effects into account of two imaginary rotlets, positioned at $(0, \pm c)$. In the case where $i = 1$, the rotlets rotates in the same direction; in the other case where $i = 2$, the rotlets rotate in opposite directions. The second contribution, ψ_i, is due to the presence of the boundaries: it indicates the effect of the boundaries on the interior flow field.

Since $\nabla^2\psi_i^{(\sigma)} = 0$, the unknown functions ψ_i are solutions of the biharmonic equation as well. The required no-slip boundary condition can be translated into Neumann boundary conditions for ψ_i. One important feature of the construction of the solution is the appearance of a system of infinite series, which is solved using the method of reduction [3]. Some mathematical background can also be found in Ref. [4].

SOME RESULTS

The velocity field $\mathbf{u} = -\nabla \times (\mathbf{e}_z \Psi)$, calculated from the overall stream function Ψ (eq.(1)), is used in a contour kinematics simulation to follow the time evolution of an initially circular blob of tracer material placed in the centre of the rectangular domain. In the two cases here presented, the individual adjustable rotlets are positioned at $(0, \pm c)$, where $c = 0.6$, and the rotlet intensity σ is set equal to 1. The main difference is the position of the walls with respect to the position of the rotlets. This makes it possible to study the effects of the walls on the interior flow field and therefore the effects of the walls on the chaotic behaviour of the tracer particles.

Rectangular domain

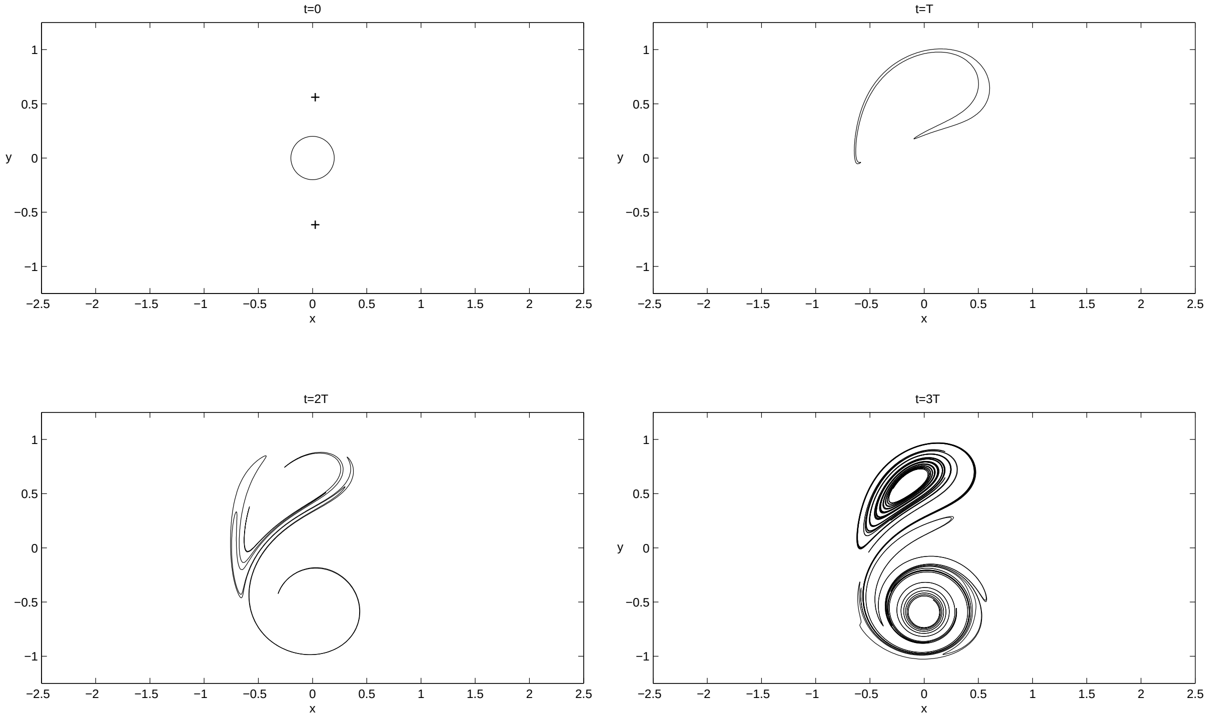

Figure 1. Advection of a patch tracer material due to two blinking rotlets ($c = \pm 0.6$, $\sigma_A = \sigma_B = 1$, $a = 2b = 2.5$).

Square domain

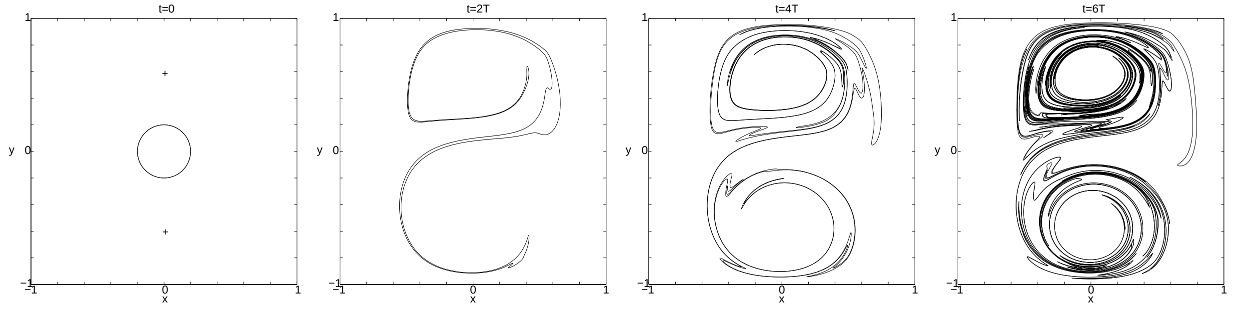

Figure 2. Advection of a patch tracer material due to two blinking rotlets ($c = \pm 0.6$, $\sigma_A = \sigma_B = 1$, $a = b = 1$).

References

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