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SENSITIVE REGIONS AND OPTIMAL PERTURBATIONS IN THE FLOATING ZONE USING THE ADJOINT SYSTEM

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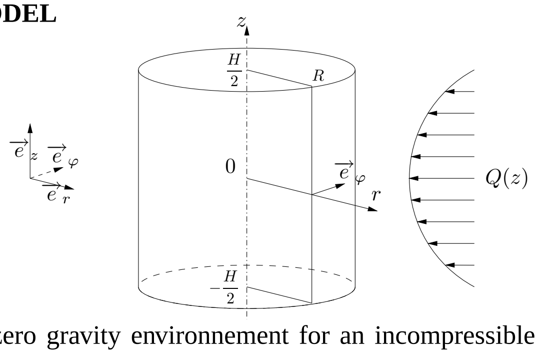
Summary Thermocapillary convection arises in free surface systems, particularly in non-isothermal liquid bridges. In this study, the mechanisms of the growth of instabilities are analysed using an adjoint technique. It allows to determine the most sensitive regions of the flow to local perturbations. The most sensitive regions to thermal perturbations are presented.

INTRODUCTION

The floating zone is a free crucible process used to produce high-quality crystals. A molten zone is created by a lateral heating between a feed and a single crystal rod, and helds by capillary forces. The translation of the material through the heat flux induces the solidification of the crystal. Temperature gradients induce surface tension variations, the source of thermocapillary convection. In order to reduce buoyancy effects, experiments have been performed in a low gravity environment and have demonstrated that thermocapillary convection alone can induce defects in the product due to flow instabilities. A major goal is to identify the mechanisms leading to the growth of those instabilities. A value of interest is the critical Marangoni number of the flow characterizing the transition to instability. The 2D axi-symmetric study of the floating zone performed by [3] has pointed out a symmetry breaking with respect to the midplane for $Pr = 10^{-2}$. The steady-state solution becomes non-symmetric via a subcritical pitchfork bifurcation, followed by a saddle-node bifurcation. This kind of flow is usually studied by direct numerical simulation, continuation method and linear stability analysis. As far as we know, this work is the first attempt to identify the region of the steady thermocapillary flow where a local disturbance has the largest response, by the use of the adjoint technique [4]. This is also the first application of this method to a highly confined geometrical configuration. The floating zone model will be presented, followed by the description of the method. A representation of the most sensitive regions to thermal perturbations are shown.

FLOATING ZONE MODEL



Our model consists of a liquid bridge between two isothermal parallel concentric rigid disks of radius R which are separated by a distance H and which presents a non-deformable heated free surface. The geometry is chosen 2D axi-symmetric in cylindrical coordinates (r, z). The parameters of the model are the Prandtl number Pr, ratio of the characteristic thermal to dynamical diffusion times fixed here to 10^{-2}, the Marangoni number Ma which characterises the thermal convective regime and the aspect ratio $A = H/2R$ fixed to 1. The Navier-Stokes equations, under the Boussinesq approximation in a zero gravity environnement for an incompressible fluid, and the heat equation are used in a non-dimensionnal form:

$$\partial_t \overrightarrow{U} + \left(\overrightarrow{U} \cdot \overrightarrow{\nabla}\right) \overrightarrow{U} = -\overrightarrow{\nabla} P + Pr \Delta \overrightarrow{U} \qquad \partial_t T + \left(\overrightarrow{U} \cdot \overrightarrow{\nabla}\right) T = \Delta T \qquad \overrightarrow{\nabla} \cdot \overrightarrow{U} = 0 \qquad (1)$$

together with the boundary conditions

$$U(r = 1, z) = 0, \qquad \partial_r W|_{r=1} = -Ma\, f_n(z) \partial_z T|_{r=1}, \qquad \partial_r T|_{r=1} = Q(z)$$

$$U(r, z = \pm A) = 0, \qquad W(r, z = \pm A) = 0, \qquad T(r, z = \pm A) = 0$$

$\overrightarrow{U} = U \overrightarrow{e}_r + W \overrightarrow{e}_z$ is the velocity vector, P the pressure and T the temperature. $Q(z) = (1 - z^2)^2$ is the nondimensionnal heat flux. $f_n(z) = (1 - z^{2n})^2$ is a regularizing function of the thermocapillary stress used to avoid a singularity at the junction between the free surface and the solid boundaries. It has been shown that this function has a low impact on the flow behaviour for $n \geq 13$ [2]. The value $n = 13$ has been chosen in this study. The adopted reference scales were based on thermal diffusion. A general notation for flow components is $\mathbb{U} = (U, W, T)$.

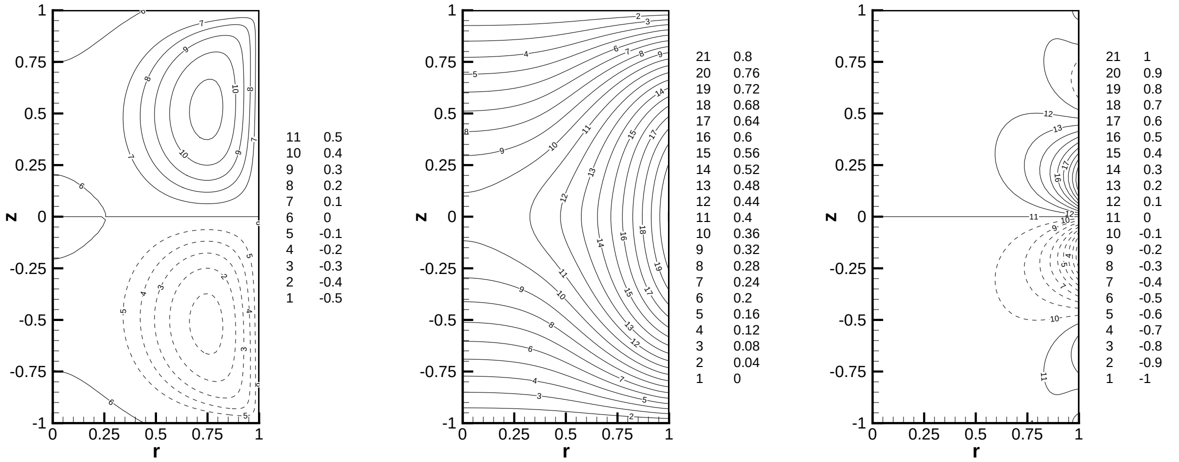
LINEARIZED AND ADJOINT SYSTEMS

Once a steady flow $\mathbb{U}_0$ has been obtained, we can determine the evolution of a small perturbation $\mathbb{u} = (u, w, \theta)$ around $\mathbb{U}_0$. The evolution equations of $\mathbb{u}$ are obtained by linearizing Eq.(1) and read $\partial_t \mathbb{u} = \mathscr{L}(\mathbb{U}_0, \mathbb{u})$. Let us assume that the linear operator $\mathscr{L}(\mathbb{U}_0, \bullet)$ can be diagonalized and, with a scalar product defined later, that the normalized complex eigenvectors $(\mathbb{u}_i)$ form a basis of the perturbation flow space. The associated eigenvalues (λ_i) are ordered first by decreasing real part and secondly by increasing imaginary part. Since $\mathscr{L}(\mathbb{U}_0, \mathbb{u}_i) = \lambda_i \mathbb{u}_i$, the evolution of a perturbation $\mathbb{u}$ such that $\mathbb{u}(t=0) = \mathbb{u}_i$ is : $\mathbb{u}(t) = \exp(\lambda_i t)\mathbb{u}_i$. If the initial perturbation $\mathbb{u}$ can be decomposed in the eigenbasis: $\mathbb{u}(t=0) = \sum_{i=1}^{+\infty} a_i \mathbb{u}_i$, its evolution is: $\mathbb{u}(t) = \sum_{i=1}^{+\infty} a_i \exp(\lambda_i t)\mathbb{u}_i$. Considering the case of a real value λ_1, we have, after a sufficiently long time $\mathbb{u}(t) \simeq a_1 \exp(\lambda_1 t)\mathbb{u}_1$. The greater is the value of $|a_1|$, the stronger is the perturbation. To calculate a_1, let us define the scalar product $\left(\mathbb{u}^{(1)}|\mathbb{u}^{(2)}\right) = \int_{-A}^{A}\int_0^1 \left(u^{(1)}\underline{u}^{(2)} + w^{(1)}\underline{w}^{(2)} + \theta^{(1)}\underline{\theta}^{(2)}\right) r\mathrm{d}r\mathrm{d}z$. The adjoint operator $\mathscr{L}^*(\mathbb{U}_0, \bullet)$ of $\mathscr{L}(\mathbb{U}_0, \bullet)$ is defined by $(\mathscr{L}(\mathbb{U}_0, \mathbb{u})|\tilde{\mathbb{u}}) = (\mathbb{u}|\mathscr{L}^*(\mathbb{U}_0, \tilde{\mathbb{u}}))$. The operator $\mathscr{L}^*(\mathbb{U}_0, \bullet)$ having eingenmodes $\left(\tilde{\lambda}_i, \tilde{\mathbb{u}}_i\right)$ such that $\forall (i,j) \in \mathbb{N}^2$, $(\mathbb{u}_i|\tilde{\mathbb{u}}_j) = \delta_{ij}$ and $\forall i \in \mathbb{N}$, $\tilde{\lambda}_i = \underline{\lambda_i}$, this leads to $a_1 = (\mathbb{u}(t=0)|\tilde{\mathbb{u}}_1)$.

Let us consider a family $\delta\mathbb{T}(r_p, z_p) = (0, 0, \delta(r - r_p)\delta(z - z_p))$ of initial local temperature perturbations. Their values of a_1 are given by: $a_1(r_p, z_p) = \int_{-A}^{A}\int_0^1 \delta(r - r_p)\delta(z - z_p)\tilde{\theta}_1 r\mathrm{d}r\mathrm{d}z = \tilde{\theta}_1(r_p, z_p) r_p$.

NUMERICAL METHOD AND RESULTS

Calculations were performed with a spectral collocation method on Gauss-Radau points along r and Gauss-Lobatto points along z, using the projection-diffusion method [1]. The steady flow is calculated with a Newton method, the first eigenmodes of $\mathscr{L}$ and $\mathscr{L}^*$ with an Arnoldi method.



Ψ_0 T_0 $r\tilde{\theta}_1/max|\tilde{\theta}_1|$

The stream function Ψ_0 and temperature T_0 are represented for the steady flow at $Ma = 106$, the critical Marangoni number being equal to 104.4. The temperature component of the first adjoint eigenmode, normalized and multiplied by r, is displayed on the right. The application of a temperature impulse has maximal efficiency when it is applied on the free surface near the mid plane.

In conclusion, the most sensitive region to thermal perturbation is located on the free surface, that is near the source of motion. It is easily accessible to the experimentation.

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