ACOUSTIC WAVE PROPAGATION THROUGH A RANDOM ARRAY OF DISLOCATIONS

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The dynamics of dislocations in interaction with acoustic waves is a problem at the root of a number of outstanding issues in the mechanics of materials. Mechanically excited phonons in interaction with dislocations appear in acoustics experiments, and the vibrating string model of dislocation damping [1] has been quite succesful in explaining a wealth of data, such as measurements of damping, internal friction and modulus change of solids. Nevertheles, there are many unexplained results, especially those obtained through thermal conductivity measurements at low temperatures and the review of Anderson [2] has highlighted the need for an improved theoretical understanding of the interaction between acoustic and elastic waves on the one hand, and dislocations on the other. Two important aspects that the vibrating string model does not take into account are the vector nature of wave and dislocation, and the collective effects arising from the presence of many obstacles along the way of wave propagation.

There are further outstanding problems in the mechanics of materials, such as the brittle to ductile transition and fatigue [3] where acoustic, or ultrasonic, measurements could lead to an improvement in their understanding. However, for this to be feasible, an improved theoretical understanding of the sound-dislocation interaction is needed. Recently it has become possible to visualize, through X-ray imaging, surface acoustic waves in interaction with dislocations[4].

With the above considerations as motivation, the purpose of the present paper is to offer results towards filling the gap concerning the understanding of the elastic wave-dislocation interaction: we consider first anti plane (resp. in plane) waves in interaction with screw (resp. edge) single dislocations, and obtain formulae for the scattering cross section in which their vector nature is considered in full. Next, these results are used in a multiple scattering perturbative treatment of the coherent behaviour of an acoustic wave that travels through a random array of dislocations, and we obtain formulae for the effective speed of wave propagation, and for the attenuation length.

Scattering of an acoustic wave by a single dislocation[5] We consider acoustic waves of wavelengths large compared to dislocation core size, so that nonlinear effects near the core can be neglected. The scattering mechanism is thus as follows: The wave hits the dislocation, and as a consequence the latter oscillates. This oscillation then generates outgoing waves in directions other than the incident direction. The challenge is to compute the scattering amplitude for an edge dislocation in interaction with an in plane wave, and to fully take into accout mode conversion between the two modes present in two dimensions: longitudinal (acoustic) and transverse (shear). The response of the dislocation to an incoming wave is described by the equations found in [6], and the re-radiated waves are computed using the formaluation of Mura [7]. The main result is a formula for the scattering amplitude in two dimensions. It is a two-by two matrix, to take into account the possible ingoing and outgoing polarizations. For example, for an incoming acoustic wave and outgoing shear wave it is

$$f_{\alpha\beta}(\theta) = \frac{\mu b^2}{2M}\frac{e^{i\pi/4}}{\sqrt{2\pi\Omega\alpha^3}}\cos 2\theta_0 \;\sin(2\theta - 2\theta_0)$$

Where α (resp β) are the speeds of acoustic (resp. shear) wave propagation, θ_0 is the angle of incidence with respect to the Burgers vector b, θ is the scattering angle, μ is the shear modulus, Ω

is the wave frequency and M is the dislocation mass.

Coherent propagation through a random array of dislocations[5] In this section we determine the coherent wave number of an acoustic wave that is multiply scattered by a random array of dislocations in two dimensions. The real part of the coherent wave number gives the effective wave velocity and the imaginary part gives the attenuation length, or mean free path. A low density of dislocations is assumed and the formalism is based on a wave equation with a right hand side term whose structure is tailored to take into account the interaction described in the previous paragraph. In the simple case of a shear wave propagating through a random array of screw dislocations with density n, the effective wave number K_β is obtained at first order in perturbation theory: $K_\beta = k_\beta(1 - (\mu n b^2)/(2M\Omega^2))$, where k_β is the unperturbed wavenumber. This provides the effective wave velocity. A computation to second order gives the attenuation length, and an generalization provides the formulae for an in-plane wave propagating through a random array of edge dislocations.

Coherent propagation in three dimensions through a random array of dislocation loops[5] Dislocation loops introduce an additional length scale—the loop size—into the problem, and the main result here is a change in the scaling of the scattering cross section: rather than increasing with wavelength, it decreases.

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