POD ANALYSIS OF COHERENT STRUCTURES IN TURBULENT FLOWS

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Abstract POD (proper orthogonal decompositions) is an efficient method to extract turbulent coherent structures. In this paper, POD technique is applied to the study of turbulent natural thermal convection between two vertical plates and the study of planar compressible mixing layer flow, based on DNS database. Both the direct POD method and the snapshots method are applied in the present work. The distribution of energy among POD modes has been closely examined. The most energetic structures are extracted, respectively spiral structures and streamwise vortexes in natural convection and span-wise vortices, turbulence structure etc in mixing layer. It has been observed that the flow structures in experiment match well with the ones in POD, and these structures are the essential characteristic of natural convection and mixing layer flow. The direct method and the snapshots method have also been compared in this paper.
Keywords Coherent structures, POD, natural convection, planar compressible mixing layer

INTRODUCTION

Though the coherent structures have been widely observed, there are many different methods to extract these structures. Among them, the Proper Orthogonal Decomposition (POD) method, which was first introduced in the fluid mechanics community by Lumley[1], has great influence in the turbulence study. From the point of energy, POD method can capture the most energetic structures that are dominant in the flow. Applying this method, many typical flows have been successfully studied such as the flow in a channel, plane boundary and Rayleigh-Benard convection. It has been proved efficient to analyze the dynamics of coherent structures. Now the POD method is applied here to identify the structures in the turbulent natural convection between two vertical plates and also the mixing layer flow.
POD method has also derived from the origin approaches to this so-called snapshots (strobes) method. Comparison of these specific processing methods is also mentioned.
(Due to the limited length of this extended summary, only the research on the structures in the turbulent natural convection between two vertical plates will be specifically described here.)

POD METHOD THEORY

POD extracts objectively optimal basis functions from experimental or DNS data to describe the characteristic of the data set. This basis is optimal in the sense that a finite number of these orthogonal modes represent more of energy than any other set of orthogonal modes.
Speaking of POD method, the origin way that Lumley introduced has met some difficulties in the determination of the basis functions from a practical point of view, incase of dealing with large-scale database. To solve this problem, Sirovich[3] brings forward a the method of snapshots or strobes. Both the two method is wildly used today in data analysis. Because of the different scale of our databases of these two different flows, we respectively applied these two methods.

Direct method

To gain the set of POD basis, an eigenvalue function is to be calculated. And this eigenfunction's order is up to $J \times N$, while J is the dim of the constants to be determined and N is the number of the components to be used as basis functions. So if a realistic count of the number functions in a realistic simulation gives a number of quite large, the caculation of eigen-problem will be out of the question.

Method of snapshots or strobes

Order of eigen-problem by means of snapshots is only correlate to the number of snapshots you chosen. We are interested in choosing a set of coherent structures that capture most of the energy. So we could confirm a minims number of snapshots considering how many energy we want to catch abstractly. The question of whether to ues the direct method or the snapshots method approach rests on a comparison of $J \times N$ and the minims number of snapshots.

DATA SET (NATURAL CONVECTION PART)

Natural convection

POD modes are obtained using 200 snapshots from a well-resolved DNS of natural convection between vertical plates at $Ra = 5.4 \times 10^5$ and $\Pr = 0.71$[2]. The computational domain is of the size $L_1 \times L_2 \times L_3 = 2\pi \times 1 \times \pi$ (gridding size $N_1 \times N_2 \times N_3 = 180 \times 48 \times 90$) in the streamwise, wall normal and spanwise directions.

Mixing layer

POD modes are obtained using 200 snapshots from a well-resolved DNS of mixing layer flow at $M_1 = 2.9, M_2 = 1.3, M_c = 0.8$, $\mathrm{Re} = 400, \mathrm{Re}_c = 525$ [4]. The computational domain is of the size $L_1 \times L_2 \times L_3 = 350 \times 120 \times 30$ (gridding size $N_1 \times N_2 \times N_3 = 875 \times 140 \times 80$) in the streamwise, wall normal and spanwise directions.

RESULT (NATURAL CONVECTION PART)

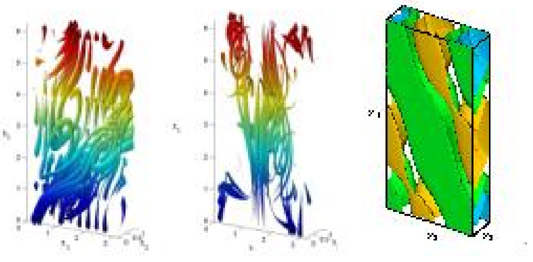
In Fig. 1 to Fig. 3, flow structures with and without mean flow are shown. Since in the real flow field the typical structures are all observed in the background of mean flow, the mode structures with mean flow are more consistent with the spiral structures observed in the DNS flow field. In Fig.2 to Fig. 3, the temperature field is displayed at the same time. It can be observed that the temperature structures match with spiral velocity structures, which implies that large-scale structures are always accompanied with heat transport. This is an essential characteristic of natural convection.

In Fig. 4, time series of coefficients associated with some modes are shown. We can see that large-scale structures, which contain more energy, have longer time scale than the small structures. It corresponds with the common sense to coherent structures.

CONCLUSIONS (NATURAL CONVECTION PART)

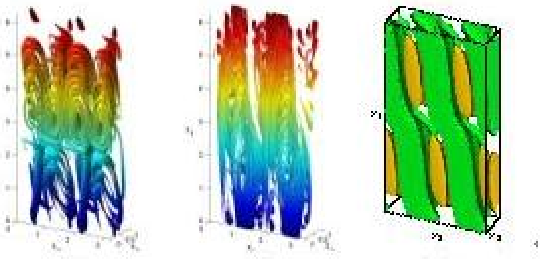
1). Via POD method, the distribution of energy can be known. The energy converge fast enough to do low dimension model research.

2). The modes implying spiral structure contain most of fluctuating energy. They are the main style of coherent structures in the turbulent natural convection between two vertical plates.



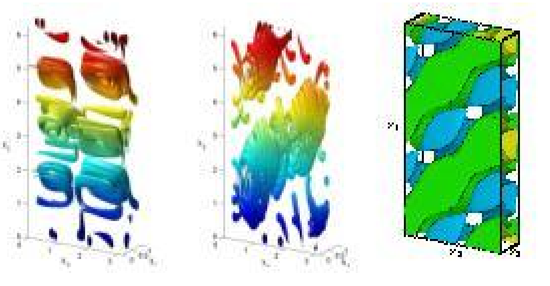
(a) (b) (c)

Fig. 1. Flow field associated with 2nd mode (a) streamtube without mean velocity. (b) streamtube with mean velocity. (c) temperature isosurface.



(a) (b) (c)

Fig. 2. Flow field associated with 4th mode (a) streamtube without mean velocity. (b) streamtube with mean velocity. (c) temperature isosurface.



(a) (b) (c)

Fig. 3. Flow field associated with 5th mode. (a) streamtube without mean velocity. (b) streamtube with mean velocity. (c) temperature isosurface. isosurface.

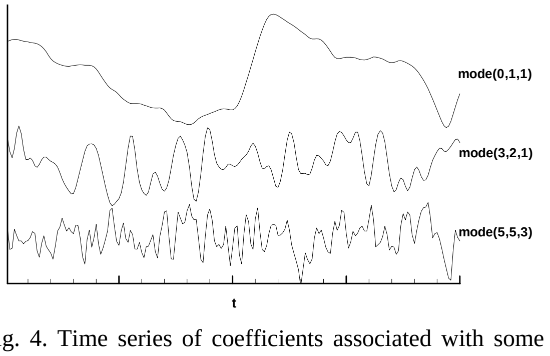


Fig. 4. Time series of coefficients associated with some typical modes.

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