TOPOLOGY OPTIMIZATION APPLIED TO THE DESIGN OF FUNCTIONALLY GRADED MATERIAL (FGM) STRUCTURES

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Summary Functionally Graded Materials (FGMs) possess continuously graded properties with gradual change in microstructure. The concept of FGM is closely related to the topology optimization concept which essentially consists in a design method that seeks a continuum optimum material distribution in a design domain. This suggests that FGM structures can be designed by using topology optimization method. Thus, in this work topology optimization method is applied to design FGM structures considering a minimum compliance design problem. The topology optimization formulation considers the so-called continuous topology optimization formulation where a continuous change of the material properties is considered inside the design. As example, a new design was obtained where the problem is considered in an FGM domain, that is, a domain where the properties change in a certain direction in according to a specified law, leading to a structure with asymmetric stiffness properties.

INTRODUCTION

Functionally Graded Materials (FGMs) possess continuously graded material properties and are characterized by spatially varied microstructures created by nonuniform distributions of the reinforcement phase as well as by interchanging the role of reinforcement and matrix (base) materials in a continuous manner. The smooth variation of properties offer advantages such as reduction of stress concentration and increased bonding strength [1]. In this paper, FGMs are studied in conjunction with the concept of topology optimization design [2]. The design of FGM structures considering topology optimization has been considered in previous works [3,4] focusing mainly on thermal and thermomechanical applications, including transient problems, by defining the design variable in a piecewise manner in the discretized domain. The objective of the present work is to design FGM structures by using the concept of *continuum topology optimization* [5]. In this method, a continuum distribution of the design variable inside of the finite element domain is considered leading to a fully continuous FGM during the design problem. As objective function, the traditional formulation for stiffness design problem is considered initially where the objective is to find the material distribution that minimizes the mean compliance. A new design of structure is obtained by considering the distribution the FGM material itself in the design domain which leads to a structure with asymmetric stiffness properties.

THEORETICAL FORMULATION

Topology Optimization Concept

The objective of topology optimization is to determine holes and connectivities of the structure by adding and removing material in the extended domain which is a large fixed domain that must contain the whole structure to be determined. A main question in topology optimization is how to change the material in a binary form (e.g. from zero to one). The use of a discrete approach would present difficulties in the numerical treatment of the problem due to multiple local minimum. Thus, a material model must be defined to allow the material to assume intermediate property values by defining a function of a continuous parameter (design variable) that determines a mixture of two materials throughout the domain. This provides enough relaxation for the design problem. In this work, the topology optimization implementation considers material models based on the so-called density methods or artificial power law approaches, which will be employed together with a filtering technique to control the mesh dependency [2].

FGM-SIMP material model

The traditional SIMP (Solid Isotropic Material with Penalization) model [2] states that in each point of the domain, the material property is given by $E^H = \rho^p E_0$, where E^H and E_0 are the Young modulus of the homogenized material and basic material that will be distributed in the domain, respectively, ρ is a pseudo-density describing the amount of material in each point of the domain which can assume values between 0 and 1, and p is a penalization factor to recover the discrete design. For ρ equal to 0 the material is equal to void and for ρ equal to 1 the material is equal to solid material. Considering the objective of designing a structure in an FGM domain where the properties change in a certain direction in according to a specified law, the property E_0 above will depend on the position **x**, e.g. defining an exponentially graded material. Thus, the new material model can be written in the following form:

$$E^H = \rho^p E_0 e^{\alpha x + \beta y} \tag{1}$$

where α and β are coefficients that define the change of material property in the domain, and x and y are the position Cartesian coordinates. This is a common situation when dealing with FGM materials [1], which is expected to result in a non-symmetric design. The domain is discretized into finite elements and the pseudo-density is defined for each node and interpolated inside the finite element [1,5].

Design Problem Formulation

The traditional formulation for stiffness design problem is considered where the objective is to find the material distribution that minimizes the mean compliance ($C_{mean}=\boldsymbol{U}^t\boldsymbol{F}$) considering a volume constraint ($\sum_{I=1}^{N}\rho_I \leq V_{des}$) and equilibrium equation ($\boldsymbol{KU}=\boldsymbol{F}$) [2], where $\boldsymbol{K}$ is the stiffness matrix, $\boldsymbol{U}$ denotes displacements, and $\boldsymbol{F}$ denotes the force vector. The design variables are the pseudo-density ρ_I, which can assume different values in each finite element node [1,5].

RESULTS

The algorithm was implemented using C language. Four node bilinear elements considering plane stress formulation are used in the finite element formulation. Since the traditional stiffness design problem is considered, the optimality criteria method [2] was applied to solve the optimization problem since it is very efficient. A graded structural design is presented in an FGM domain, that is, a domain where the properties of the basic material change in a certain direction according to equation (1). The idea is to obtain a structure with asymmetric stiffness properties. The normalized Young`s modulus E_0 is equal to 10. The design domain considered and corresponding applied distributed load is shown in Figure 1a. A volume material constraint of 30% and a filter with radius equal to 1.5 was considered. Figures 1b and 1c illustrate the designs obtained considering the material variation in the x ($\beta=0$) direction. Notice the significant change of the resulting topology as a function of the material gradient.

a) b) c) d)

Figure 1: a) Initial design domain (50 x 50 mesh); b) Topology obtained using the traditional SIMP material model; c) Same using FGM-SIMP material model in the x direction with α=0.04, and filter (radius equal to 1.5); d) same with α=0.06.

CONCLUSIONS

Continuum topology optimization [5] was successfully applied to design FGM structures in which the material properties change within the design domain in a continuous manner. New designs were obtained where the distribution of the FGM itself is considered in the design domain, resulting in a structure with asymmetric stiffness properties. Thus, novel structural types are obtained by exploring the functionally graded materials idea. As future work, the design of composite unit cells made of FGMs will be considered using continuum topology optimization concepts.

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