Simulation of planar wave instabilities in liquid-fluidized beds

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It is well known that dense fluidized beds exhibit a rich variety of complex, inhomogeneous flow structures, ranging from one-dimensional traveling waves to bubble-like voids, and the hierarchy of these structures has been a subject of many theoretical and experimental studies [1, 2]. An Eulerian two-phase flow model, which treats the fluid and particle phases as interpenetrating continua, coupled with simple phenomenological closures for the effective stresses and the fluid-particle interaction force, seems to capture the experimentally observed structures in a qualitatively correct manner; however, quantitative predictions remain elusive [1]. Much effort is in progress to develop more accurate closures through computational simulations of flows involving homogeneous assemblies of particles [e.g., see 3, 4].

In the present study, we first demonstrate that the transition from homogeneous fluidization (worming regime) to an inhomogeneous system with traveling, one-dimensional waves observed experimentally in fluidized beds of limited lateral extent at relatively low fluidization velocities [2] can be captured through detailed discrete particle simulations. From our simulations, we then determine how the particle concentration, fluid and particle velocities, fluid-particle interaction force, stress due to particle-particle interactions, fluid phase viscous and pseudo-turbulent stresses vary in the traveling wave. Such detailed information not only allows us to examine the importance of various terms in the continuum model and various microphysics such as lubrication force, but also assess whether closures developed from simulations of homogeneous suspensions should be supplemented for non-local effects in inhomogeneous systems.

The flow domain that we consider in our simulations is fully periodic. It contains spherical, solid particles (all having the same diameter d_p). The size of the domain in the examples illustrated below is $6d_p$ in the two lateral directions, and $20d_p$ in the flow direction. A body force exerted on the fluid drives the flow. This body force is balanced by a force acting on the solids in the opposite direction. We apply lattice-Boltzmann discretization of the Navier-Stokes equations to solve the liquid flow in between the particles [5]. The particles (which were given a diameter of 10 grid-spacing) are represented by means of an immersed boundary technique [6]. Their motion is controlled by the hydrodynamic force emerging from the lattice-Boltzmann method, by the leading order term of the radial lubrication force that acts as a sub-grid force when particle surfaces are in close proximity (less than one grid-spacing apart), and by hard-sphere collisions (that in this study are assumed to be elastic, and frictionless).

The operating conditions can be fully characterized by three dimensionless parameters: the average solids volume fraction ϕ_0, the density ratio ρ_s/ρ_f, and the Reynolds number of a single particle, freely falling in the working fluid: $Re_t = \frac{u_t d_p}{\nu}$. These three parameters are used to translate physical units into lattice-Boltzmann units. The simulations are designed to mimic one of the cases defined by Duru et al. [2], more specifically their Combination #7 (with ϕ_0=0.488, Re_t=120 and ρ_s/ρ_f =4.1).

In Figure 1, we show snapshots of our flow system operating in the worming (1A), and in the planar wave regime (1B). In order to reach the latter regime, we had to operate the bed at a significantly lower volume fraction compared to the experiment (0.43 versus 0.488). This is most probably due to the hydrodynamic diameter of the particles being larger than their given diameter on which ϕ_0 is based [7]. The space-time graphs in Figure 2 show the transition from the worming to the planer-wave regime. The homogeneous cases (A and B) have much less volume fraction fluctuations than the other cases that are in the planar wave regime. The wave speed that can be derived from the space-time graphs corresponds

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very well with the experimentally observed wave speed: $0.24u_t$ in the experiment, $0.25u_t$ in the simulation. Time-averaged waveforms are given in Figure 3. These waves are quite noisy, and the simulation must be run for a much longer duration to obtain a smoother one-dimensional wave. Comparison of Figures 1D and 1E, and the waveforms in Figure 3 suggest that the lubrication force is not important in this example.

In this presentation, we will describe and discuss further details concerning spatial variation of various quantities such as fluid-particle interaction force and particle phase pressure.

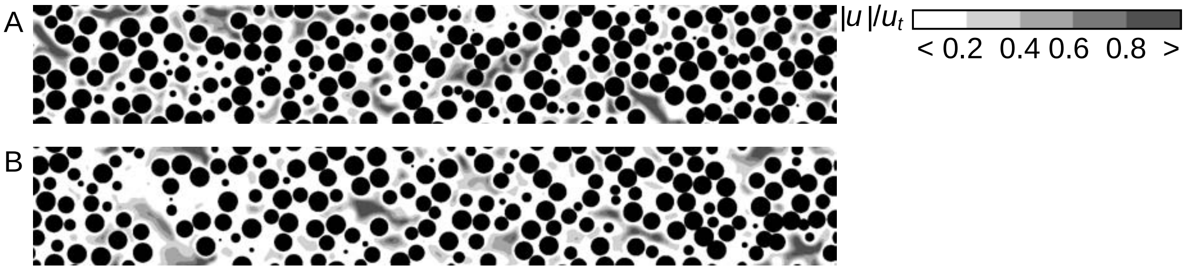

Figure 1. Cross-sections of single realizations of the solid-liquid flow. In both cases Re_t=120 and ρ_s/ρ_f =4.1. Case A has ϕ_0=0.48, case B has ϕ_0=0.41. The gray scale indicates the absolute value of the liquid velocity.

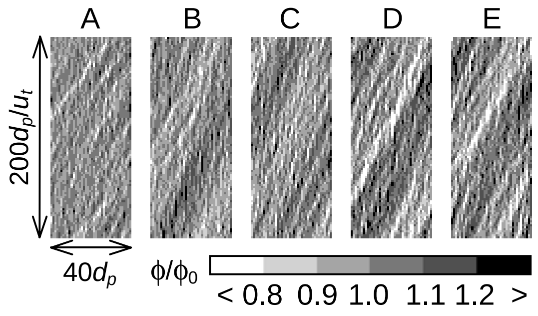

Figure 2. Space (horizontal) – time (vertical) plots of the solids phase volume fraction. Case A-D have ϕ_0=0.48, 0.46, 0.43, and 0.41 respectively, and do not take into account the radial lubrication force. Case E is the same as case D, but now with lubrication.

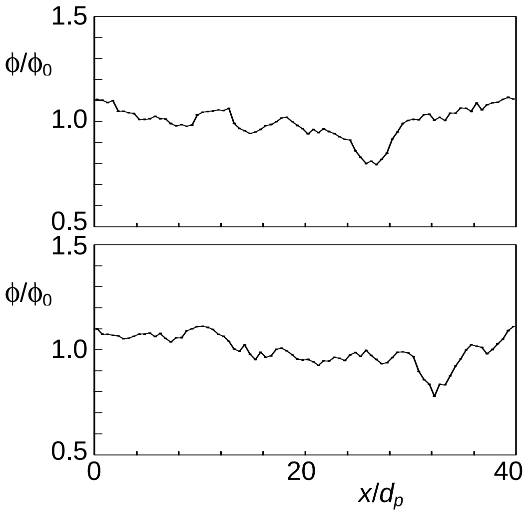

Figure 3. Time-averaged wave form of cases D (bottom) and E (top) as defined in Figure 2.

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