NONLINEAR DYNAMICS OF AXIALLY MOVING VISCOELASTIC STRINGS BASED ON TRANSLATING EIGENFUNCTIONS

Li-Qun Chen , Neng-Hui Zhang

Department of Mechanics, Shanghai University, Shanghai 200436, China

Summary Nonlinear dynamics is investigated for transverse vibration of axially moving strings. The Kelvin viscoelastic model is chosen to describe the viscoelastic property of the string material. The tension is characterized as a small periodic perturbation on a constant mean value. The translating string eigenfunctions are employed to discretize the governing equation, a nonlinear partial differential equation. By use of the Poincaré maps, the dynamical behaviors are identified based on the numerical solutions of the ordinary differential equations that define respectively the 1, 2, 3 and 4-term truncated systems. The bifurcation diagrams are calculated in the case the dynamic viscosity is varied while other parameters are fixed. The bifurcation diagrams of 1, 2, 3 and 4-term truncated systems are qualitatively same. The numerical results indicate that chaos occurs for the small dynamic viscosity, and regular and chaotic motions alternately appear for the increasing dynamic viscosity.

INTRODUCTION

Many engineering devices involve the vibrations of axially moving strings. Traditionally, the investigations on axially moving strings were focused on equilibriums and periodic motions [1]. Recently, there are some researches on chaos and bifurcation of axially moving strings[2-4]. However, all these researches discretized the governing equation based on the stationary string eigenfunctions. The present work applies the translating eigenfunctions [5], which has the good convergence properties, to discretize the governing equation.

DISCRETIZATION BASED ON THE TRANSLATING EIGENFUNCTIONS

The dimensionless form of the governing equation for transverse vibration of an axially moving viscoelastic string is[2]

$$\frac{\partial^2 v}{\partial\tau^2}+2\gamma\frac{\partial v}{\partial\tau\partial\xi}+\left(\gamma^2-1-\alpha\cos\omega\tau\right)\frac{\partial^2 v}{\partial\xi^2}-\frac{3E_\mathrm{e}}{2}\left(\frac{\partial v}{\partial\xi}\right)^2\frac{\partial^2 v}{\partial\xi^2}-\frac{E_\mathrm{v}}{2}\frac{\partial}{\partial\tau}\left(\frac{\partial v}{\partial\xi}\right)^2\frac{\partial^2 v}{\partial\xi^2}+E_\mathrm{v}\frac{\partial v}{\partial\xi}\frac{\partial}{\partial\tau}\left(\frac{\partial v}{\partial\xi}\frac{\partial^2 v}{\partial\xi^2}\right)=0 \tag{1}$$

where $v(\xi,\tau)$ is the transverse displacement at time τ and axial coordinate ξ, γ is the axially moving speed, α and ω are respectively the amplitude and frequency of the periodic tension perturbation, and E_e and E_v are respectively the stiffness constant and the dynamic viscosity in the Kelvin model of the viscoelastic string, all in the dimensionless form.

Equation (1) can be cast into the state variable form

$$\boldsymbol{A}\frac{\partial\boldsymbol{w}}{\partial\tau}+\boldsymbol{B}\boldsymbol{w}-\boldsymbol{N}\boldsymbol{w}=0 \tag{2}$$

where

$$\boldsymbol{w}=\begin{Bmatrix}u\\ v\end{Bmatrix}=\begin{Bmatrix}\dfrac{\partial v}{\partial\tau}\\ v\end{Bmatrix},\ \boldsymbol{A}=\begin{Bmatrix}1 & 0\\ 0 & \left(\gamma^2-1\right)\dfrac{\partial^2}{\partial\xi^2}\end{Bmatrix},\ \boldsymbol{B}=\begin{Bmatrix}2\gamma\dfrac{\partial}{\partial\xi} & \left(\gamma^2-1\right)\dfrac{\partial^2}{\partial\xi^2}\\ \left(\gamma^2-1\right)\dfrac{\partial^2}{\partial\xi^2} & 0\end{Bmatrix}$$

$$\boldsymbol{N}=\begin{Bmatrix}E_v\left(\dfrac{\partial v}{\partial\xi}\right)^2\dfrac{\partial^2}{\partial\xi^2} & \left[\alpha\cos(\omega\tau)+\dfrac{3}{2}E_e\left(\dfrac{\partial v}{\partial\xi}\right)^2+2E_v\dfrac{\partial v}{\partial\xi}\dfrac{\partial u}{\partial\xi}\right]\dfrac{\partial^2}{\partial\xi^2}\\ 0 & 0\end{Bmatrix} \tag{3}$$

Under the homogeneous boundary condition, the solution of Equation (2) can be assume as the expansion

$$\boldsymbol{w}=\begin{Bmatrix}u\\ v\end{Bmatrix}=\begin{Bmatrix}\sum_{n=1}^{\infty}\omega_n\left[-q_n^\mathrm{R}(\tau)\psi_n^\mathrm{I}(\xi)+q_n^\mathrm{I}(\tau)\psi_n^\mathrm{R}(\xi)\right]\\ \sum_{n=1}^{\infty}\left[q_n^\mathrm{R}(\tau)\psi_n^\mathrm{R}(\xi)+q_n^\mathrm{I}(\tau)\psi_n^\mathrm{I}(\xi)\right]\end{Bmatrix} \tag{4}$$

where and q_n^R are q_n^I components of the generalized coordinates, and the translating eigenfunctions, which are orthogonal with respect to both operator matrix $\boldsymbol{A}$ and $\boldsymbol{B}$, are [5]

$$\psi_n^\mathrm{R}(\xi)=\frac{1}{n\pi}\sqrt{\frac{2}{1-\gamma^2}}\sin(n\pi\xi)\cos(\gamma n\pi\xi),\ \psi_n^\mathrm{I}(\xi)=\frac{1}{n\pi}\sqrt{\frac{2}{1-\gamma^2}}\sin(n\pi\xi)\sin(\gamma n\pi\xi) \tag{5}$$

Substituting Equation (4) into Equation (2), taking the inner product of both hands of the resulting equation, using the orthonormality of the eigenfunctions, and retaining only the first m terms in the resulting equation yield the m-term truncated system

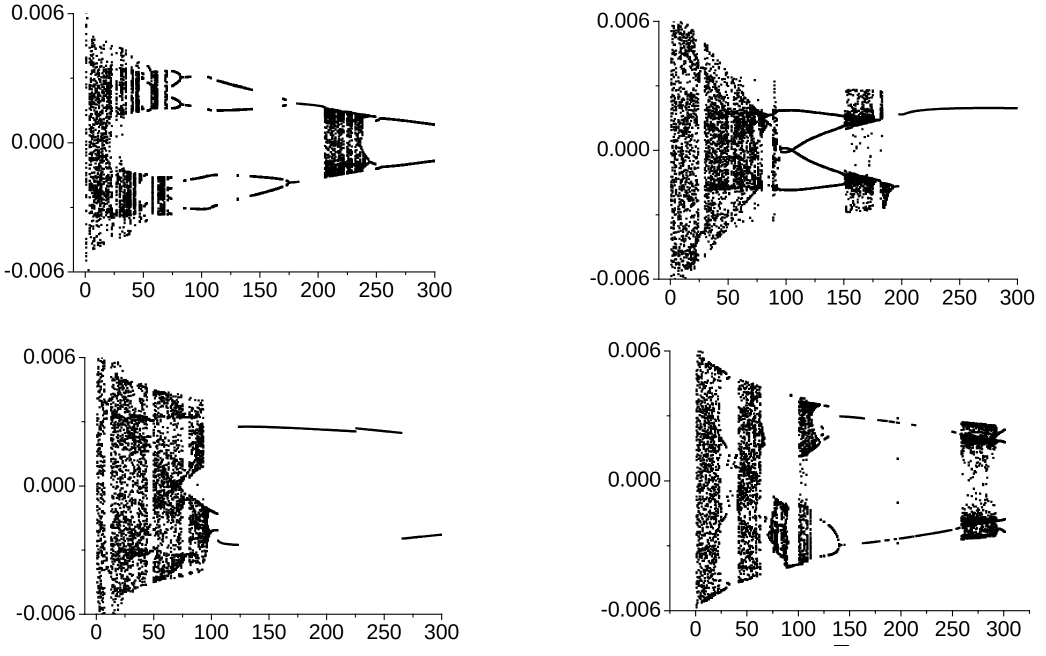
$$\dot{q}_i^{\mathrm{R}}(\tau)=\omega_i q_i^{\mathrm{I}}(\tau)+\sum_{n=1}^{m}\left[q_n^{\mathrm{R}}(\tau)c_{ni}^{\mathrm{RR}}(\tau)+q_n^{\mathrm{I}}(\tau)c_{ni}^{\mathrm{IR}}(\tau)\right],\ \dot{q}_i^{\mathrm{I}}(\tau)=-\omega_i q_i^{\mathrm{R}}(\tau)+\sum_{n=1}^{m}\left[q_n^{\mathrm{R}}(\tau)c_{ni}^{\mathrm{RI}}(\tau)+q_n^{\mathrm{I}}(\tau)c_{ni}^{\mathrm{II}}(\tau)\right] \tag{6}$$

where (a and b stand for R and I)

$$c_{ni}^{\mathrm{ab}}(\tau)=\left\langle \boldsymbol{N}\boldsymbol{\phi}_n^{\mathrm{a}}(\xi),\boldsymbol{\phi}_i^{\mathrm{b}}(\xi)\right\rangle,\ \boldsymbol{\phi}_n^{\mathrm{R}}(\xi)=\begin{Bmatrix}-\omega_n\psi_n^{\mathrm{I}}(\xi)\\ \psi_n^{\mathrm{R}}(\xi)\end{Bmatrix},\ \boldsymbol{\phi}_n^{\mathrm{I}}(\xi)=\begin{Bmatrix}\omega_n\psi_n^{\mathrm{R}}(\xi)\\ \psi_n^{\mathrm{I}}(\xi)\end{Bmatrix} \tag{7}$$

NEMERCAL RESULTS

The Poincaré map is a convenient tool to identify the dynamical behavior, especially chaos. To view globally over a range of parameter values, the bifurcation diagrams are computed. The Poincaré maps of the dimensionless displacement of the center of the moving string, which is obtained by numerical integration of Equation (6) for m=1,2,3,4 respectively. The bifurcation diagrams of the dimensionless center displacement of the string against the dimensionless dynamic viscosity E_{V} are shown in the following fingers.



CONCLUSIONS

This paper treats nonlinear dynamical behaviors of transverse vibration of an axially moving strings constituted by the Kelvin viscoelastic. The string is subject to the pulsating tension. The governing equation is discretized based on the translating string eigenfunctions For the 1, 2, 3 and 4-term truncated systems. The bifurcation diagrams of the Poincaré maps are calculated for varying the dynamic viscosity. The bifurcation diagrams of 1, 2, 3 and 4-term truncated systems are qualitatively same. The numerical results indicate that chaos occurs for the small dynamic viscosity, and regular and chaotic motions alternately appear for the increasing dynamic viscosity.

Acknowledgments

The work was supported by the National Natural Science Foundation of China (Project No. 10172056).

References

[1] Chen L.-Q.: Analysis and control of transverse vibrations of axially moving strings. *Appl. Mech. Rev.*, in press

[2] Chen L.-Q., Zhang N.-H., Zu J. W.: The regular and chaotic vibrations of an axially moving viscoelastic string based on 4-order Galerkin truncation. *J. Sound & Vib.*, **261**: 764-773, 2003.

[3] Chen L.-Q., Wu J., Zu J. W.: Asymptotic nonlinear behaviors in transverse vibration of an axially accelerating viscoelastic string. *Nonlinear Dyn.*, **35**: 347-360, 2004.

[4] Chen L.-Q., Wu J., Zu J. W.: The chaotic response of the viscoelastic traveling string: an integral constitutive law. *Chaos, Solitons & Fractals*, 21: 349-357, 2004.

[5] Wickert J. A., Mote C. D. Jr.: Classical vibration analysis of axially moving continua. *J. Appl. Mech.*, **57**: 738-744, 1990