RESONANT INTERACTIONS OF 3D INSTABILITY WAVES IN AN AIRFOIL BOUNDARY LAYER FOR HARMONIC AND BROADBAND PERTURBATIONS

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Summary This paper is devoted to a systematic experimental investigation of resonant interactions of Tollmien-Schlichting (TS) waves occurred at weakly-nonlinear stages of the laminar-turbulent transition in an essentially non-self-similar boundary layer on an airfoil with a long laminar run. The paper is aimed to analyze the importance of the TS-wave interactions in dependence on the base-flow and disturbance parameters in order to clarify two main questions: (i) how the base flow non-uniformity influence the efficiency of the resonant interaction and (ii) when these interactions are significant for the transition prediction.

MOTIVATION OF THE STUDY

For the design of subsonic, natural laminar flow airfoils usually some half-empirical transition prediction methods are applied, which are based on the linear stability theory (the *N*-factor method for instance). This approach gives reliable results as long as the streamwise extent of the nonlinear disturbance growth is short in comparison with the linear part. However, previous experimental and theoretical studies (see e.g. [1]) have shown that even at rather low TS-wave amplitudes (0.1% or even lower) the linear disturbance growth can be modified quite significantly by some resonant interactions of instability waves. In this case the application of the transition prediction methods based purely on the linear stability theory becomes questionable and some nonlinear interactions of instability waves have to be taken into account. The present paper is devoted to the detailed experimental study of resonant weakly-nonlinear interactions of TS-waves occurred in a specially designed 2D airfoil with a long laminar run where the boundary layer is essentially non-self-similar.

EXPERIMENTAL SETUP AND REGIMES OF MEASUREMENTS

Experimental model and base flow

The experiments were conducted in the Laminar Wind Tunnel of the IAG. The lower surface of the WW03BL106 airfoil section was specially designed to provide a constant threshold of the *N*-factor of $N = \ln[A(f)/A_o(f)] = 6$ at an angle of attack of 2 degree and the chord Reynolds number of 700000. All necessary base flow parameters were measured in detail by means of pressure taps and hot-wire anemometry. Very good agreement with calculations was found. The boundary layer under investigation was essentially non-self-similar at its initial part and the corresponding shape factor H_{12} decreased downstream within the region of main measurements from 2.9 to 2.6. The mean flow stability characteristics measured for both 2D and 3D TS-waves in a range of frequencies from 200 to 600 Hz, were found to agree very well with the linear stability theory.

Excitation of disturbances

The experiments were carried out at controlled disturbance conditions. The TS-waves were excited in the boundary layer by a specially designed slit disturbance source, which was mounted flush with the surface at $s/s_{max} = 0.13$ (where $s_{max} = 604$ mm is the arclength measured from the leading edge). The slit was 0.2 mm wide and extended 290 mm in spanwise direction. Below the slit, 116 equally spaced pneumatic tubes were connected to 32 micro loudspeakers, which were driven by power amplifiers and a 16-channel signal generator. This disturbance source enabled to generate a great variety of initial disturbance spectra consisted of various combinations of 2D and 3D TS-waves with desirable frequencies and spanwise wavenumbers, including broadband perturbations.

Main regimes of measurements

In total, several dozens of initial disturbance spectra (and regimes of excitation) were examined. The following most important regimes have been studied. First of all the resonant interactions in symmetric tuned TS-wave triplets were investigated. Every triplet consisted of a 2D fundamental wave and a pair of 3D subharmonic waves with amplitudes of about one order lower than the fundamental one. The dependence of the tuned resonance on the fundamental wave frequency and amplitude was studied in a wide range, as well as the dependence on the initial phase shift between the fundamental wave and the subharmonic-wave pair, which was measured either directly or by means of a method of 'extremely small' (–1%) subharmonic frequency detuning. The influence of frequency detunings Δf_s of the subharmonic-wave frequency f_s was examined in a range of detuning parameter $\Delta f_s/f_s$ from –80 to +80% for four different frequencies of the 2D fundamental wave, when the 3D quasi-subharmonic wave pairs were excited at spanwise wavenumbers, which corresponded to the resonance condition at the position of the disturbance source. The influence of the spanwise wavenumber detunings was investigated for two fundamental frequencies. The case of the strongest resonant interaction (found for the highest, of studied, fundamental frequency) was measured in detail in a range of spanwise wavenumber detunings $\Delta\beta/\beta_r$ from –100 to +100% (where β_r is the resonant spanwise wavenumber at the disturbance source location). A case of a joint interaction of a quasi-subharmonic 3D wave pair with two 2D fundamental waves was studied for fundamental-wave frequencies f_{f1} = 610 Hz and f_{f2} = 510 Hz, while the quasi-subharmonic modes had a frequency of f_s = 311 Hz. This case is regarded as an important first step for investigation of resonant interactions of continuous-spectrum (broadband) disturbances, which would simulate the conditions of a "natural" boundary-layer transition. A first case of such broadband perturbation was investigated as well. For most of the studied cases, separate development of subharmonic wave pairs in absence of the fundamental wave was also investigated for comparison. All measurements were performed for TS-wave maxima in their wall-normal profiles. The shapes of wall-normal and spanwise distributions of the disturbance amplitudes and phases were studied as well.

MOST IMPORTANT RESULTS

Tuned resonant triplets

It is found, in particular, that similar to the self-similar boundary layers (see e.g. [1]) the resonant amplification in tuned wave triplets is the double-exponential but depends significantly on the subharmonic/fundamental phase shift. A resonant suppression of the subharmonic-wave linear growth is observed when this shift is orthogonal to the resonant one. In contrast to the self-similar flows, the efficiency of the resonant interaction turned out to be strongly dependant on the fundamental-wave frequency. The interaction is very strong at high frequencies and is nearly absent at very low frequencies. This fact seems to be explained by a streamwise variation of the TS-wave dispersion characteristics associated with the base flow non-uniformity.

Frequency- and wavenumber-detuned resonances

It is found that the subharmonic-type resonant interactions have very large spectral width in the frequency spectrum and in the spanwise-wavenumber spectrum. The frequency detuned resonances turned out to be weaker (in general) than the tuned ones at the highest of studied frequency. However, for lower frequencies the strongest resonant interactions were observed in the wave triplets having positive frequency detunings with the detuning parameter $\Delta f_s/f_s$ varied from nearly zero (for fundamental frequency f_f = 610 Hz) to approximately +80% for f_f = 255 Hz. A 'cumulative effect' was found for a set of frequency detuned resonances, which led to a concentration of efforts of resonant interactions in several different wave triplets (with different fundamental frequencies) in a narrow range of subharmonic frequencies. The resonance turned out to amplify always two quasi-subharmonic modes: the excited one (f_{sexc}) and a symmetric mode (with frequency $f_{ssym} = f_f - f_{sexc}$). It is found that the mode, which has the lowest frequency (either f_{sexc} or f_{ssym}) reaches always the largest amplitudes in the end of the parametric stage of the resonant interaction. This effect is observed for all studied subharmonic-frequency detunings and fundamental-wave frequencies, regardless of the presence or absence of the predominance of frequency detuned resonances over the tuned ones. The resonances with spanwise-wavenumber detunings turned out to be weaker, in general, than those in the tuned case. At the same time, the subharmonic growth rates were found to depend significantly on the streamwise position and changed together with the base flow characteristics. In particular, the growth rate observed for $\Delta\beta/\beta_r$ = –54% close to the source was the same as that for $\Delta\beta$ = 0, while in the end of the region of measurements the strongest amplification was found for $\Delta\beta/\beta_r$ = +50%. The resonant interaction with the 2D subharmonic wave ($\Delta\beta/\beta_r$ = –100%) is found to be very weak and the measured subharmonic growth corresponded to that due to the linear instability mechanism.

Multi-wave and broadband resonant interactions

It is found that in the case of joint resonant interaction of a quasi-subharmonic wave pair with two fundamental waves a superposition principle is satisfied approximately (in a certain sense) for efforts of these two resonances and the cumulative effect leads to a very rapid growth of quasi-subharmonic waves with increments, which are as large (or even somewhat greater) as those observed in the same two resonances when they occur separately. This property leads to a very strong resonant interaction of broadband instability waves observed in a case of a white-noise-like initial spectrum simulating a kind of "natural transition". In this case a rapid resonant growth of very-low-frequency disturbances was observed, while in absence of the interaction these perturbations attenuated in agreement with the linear stability theory.

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