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FULLY NONLINEAR GLOBAL MODES AND TRANSITION TO TURBULENCE IN OPEN FLOWS

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Summary The fully nonlinear theory of global modes in open flows, proposed in recent analyses of amplitude equations, is extended to the case of Navier-Stokes equations using direct numerical simulations. The basic flow under consideration is in a first time, a parallel wake in a finite domain generated by imposing the wake profile at the inlet boundary and by adding a body force to compensate the basic flow diffusion. The link between the global bifurcation, the absolute or convective nature of the local linear instability, and the theory of speed selection for the front separating an unperturbed domain of the flow from a fully saturated solution is elucidated. In particular, thanks to the parallelism of the flow, the bifurcation scenario and the associated scaling laws for the frequency, the healing length, and the slope at the origin predicted by a previous analysis of amplitude equations are recovered with amazing precision. Non-parallel effects are then discussed. Furthermore, new scenarii involving secondary absolute instability are proposed and compared to the dynamics of mixing layers simulations conducted in the same spirit as for the wake.

Mixing layers, jets, wakes, boundary layers over wings or rotating disks, Poiseuille and Couette flows are examples of open shear flows encountered in many industrial or geophysical situations. These flows develop spatially under the combined action of advection and instabilities and eventually undergo a transition to turbulence.

In the eighties [1], the linear concepts of absolute and convective instability succeeded in predicting some aspects of open shear flow dynamics, but a description of their spatio-temporal development including nonlinear effects and secondary instabilities was lacking and even the very fact that a linear criterion describes so well strongly nonlinear flows remains mysterious.

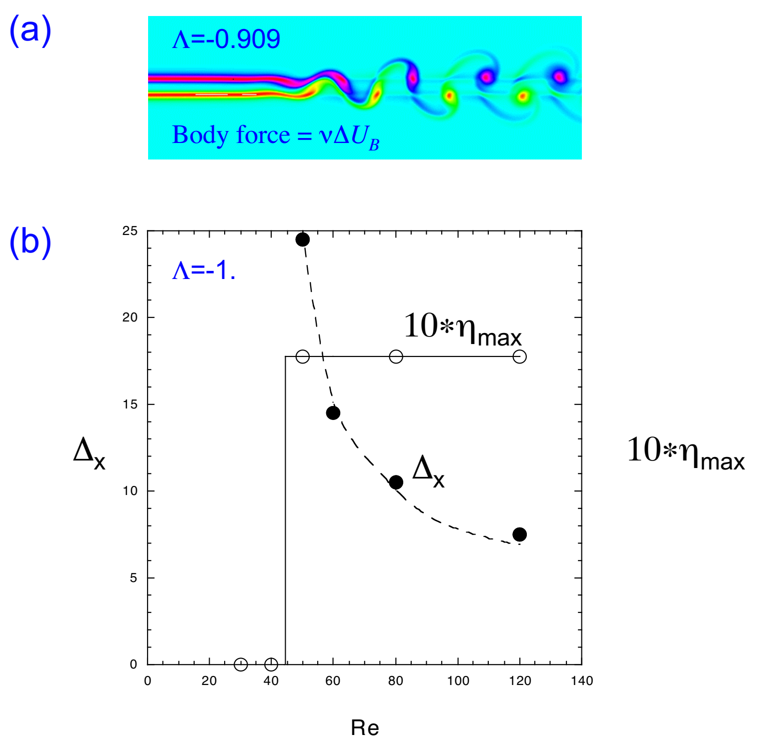

Figure 1. (a) Global Mode of the parallel wake obtained for a Reynolds number $Re = 400$ above the absolute instability threshold, for $\Lambda = -0.909$, $\Lambda = (U_0 - U_\infty)/(U_0 + U_\infty)$ where U_0 is the centre line velocity, U_∞ is the velocity at $y = \infty$. The computational box is 16.8 large and 51.2 long, the wake width being unity.(b) Bifurcation diagram for the wake, showing, for $\Lambda = -1$ (zero backflow) the saturation local enstrophy η_{max}. Following Monkewitz (1988) [9] the instability becomes absolute for Re larger than $Re_A \sim 45$. We observe that, indeed, when the instability becomes absolute the amplitude of the Global Mode measured by η_{max} suddenly increases from zero to a constant finite value as predicted by [8]. The distance Δ_x at which saturation occurs, defined by the location where $\eta(x) = 1$, is also plotted and it is well fitted by $\Delta_x \sim 60.2 * (Re - Re_A)^{-1/2}$ as predicted by the theory.

The present work reports on very recent progress elucidating open shear flow dynamics. By taking advantage of the addition of a body force that compensates the diffusion of the basic flow and makes the flow parallel in the absence of perturbations, we have computed the dynamics of a wake flow and of a mixing layer directly generated at the inlet of the domain [2]. We have established the link between the occurrence of a self-sustained oscillation (a global mode), the linear and the nonlinear transition to absolute instability [3]. First we have shown that the front separating the saturated wake solution from the basic state propagates at the speed of the edge of the linear wave packet [2, 4]. The nonlinear absolute instability as defined by Chomaz (1992) [3] coincides, therefore, with the linear absolute instability for this particular flow. Then, when a homogeneous inlet condition is applied at $x = 0$ (i.e. when the perturbations are assumed to vanish at

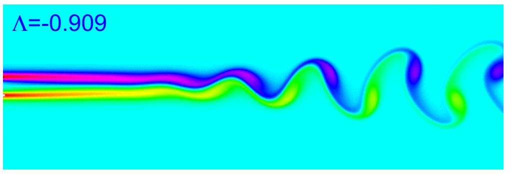

Figure 2. Global Mode of the nonparallel wake obtained for a Reynolds number $Re = 800$ for $\Lambda = -0.909$, i.e. when the inlet flow is everywhere positive. This Reynold number is well above the absolute instability threshold $Re_A \sim 125$ for this particular value of Λ. The computation box is 16.8 large and 51.2 long, the wake width being unity.

$x = 0$) we have shown that the self-sustained oscillation, the nonlinear global mode, is triggered when the flow becomes absolutely unstable. In all the cases studied, the front speed is first shown to be equal to the linear wave packet edge (pulled fronts) [2, 4, 5, 6]. Then, when inlet conditions are introduced a fully nonlinear global mode is shown to appear when the flow is absolutely unstable in the reference frame singled out by the inlet condition (figure 1). The global mode oscillates then at the absolute frequency and is made of a front stop at some distance from the inlet. Both for the wake and the mixing layer, the distance at which saturation occurs, varies as the square of the inverse of the criticality (the departure from threshold) (figure 1b).

For wakes or mixing layers, the parallel basic flow approximation is valid only if the viscosity is nil or compensated by an artificial body force. But in numerical or laboratory experiments, viscosity is finite and the basic flow imposed at the inlet evolves downstream. When the non-parallelism of the real flow is weak enough, i.e. when the scale on which the basic flow varies is large enough compared to the typical scales of the instability (the wavelength, and/or the inverse of the spatial growth rate), the present results may be extended in a straightforward manner. In that case, a nonlinear self-sustained oscillation occurs as soon as a finite domain of absolute instability is present and it is made up of a front located at the most upstream border of the absolutely unstable domain. The oscillation frequency of the Global Mode is then the absolute frequency at the front location. This border may be either a physical boundary of the flow (see [6] for a review) or a location where the flow goes from absolutely unstable to convectively unstable as explained in [7, 8]. Figures 2 shows an example of nonlinear Global Mode in a weakly non-parallel wake obtained by turning on the diffusion of the basic flow in the model. For the wake with co-flow, the instability threshold is postponed to larger values of the Reynolds number but the frequency selection is unchanged.

Since a global mode in a semi-infinite parallel flow reaches finite amplitude saturation at threshold it may itself be unstable. This will occurs when the secondary instability of the uniform saturated wave that follows the front is absolute. But the secondary instability may be absolute either before or after the primary. In the first case a stable global mode will appear first and then, if the control parameter is further increased, it may be destabilized if the secondary instability becomes absolute. In the second case the global mode is already unstable at threshold and an oscillatory or even a chaotic state appears after a single bifurcation. This second scenario involving secondary absolute instability may explain a one step transition to disorder or turbulence as observed for example in rotating disk or mixing layers experiments.

It is worth mentioning that other scenarii should exist that involve a pulled front and for which linear theory should not give a correct prediction. Since the numerical technique introduced by Delbende & Chomaz (1998) [5], allows the direct determination of the linear and nonlinear absolute or convective nature of instabilities, investigations of many different flows are now at hand. Hopefully, some of them will exhibit a nonlinear front velocity selection (pulled front) for which strong departure from linear prediction has already been demonstrated on model equations. In fluid, a nonlinear absolute instability was observed recently for the shear instability in an Hele-Shaw cell [7], the other observations being presently limited to chemical reactions [8].

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