LOW-REYNOLDS-NUMBER MOTION OF A DROP BETWEEN TWO PARALLEL PLANE WALLS

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Summary The motion of a deformable drop between two parallel plane walls in Poiseuille flow at low Reynolds number is examined using a novel boundary-integral method (BIM). Three-dimensional results are presented for neutrally-buoyant drops of arbitrary fluid-to-drop viscosity ratio, drop size, and position within the channel.

INTRODUCTION

An emulsion consists of drops of one fluid dispersed in a second, immiscible fluid. Many applications, found in areas of biotechnology, food science, geology, and pharmaceutical processing, among others, involve the flow of emulsions through pipes, channels, or porous media. When the droplet and container or pore sizes are comparable, effective-fluid or volume-averaging schemes fail, which provides motivation for developing microstructural flow models.

In the present work, the motion of a deformable drop through another immiscible fluid between two parallel walls is examined. Literature addressing the motion of a single or collection of drops using BIMs are plentiful[1,2]. Previous works give results for a two-dimensional drop in an inclined channel[3] and an array of drops through a cylindrical tube[4]. Closely related to the present work is a study of the motion of a rigid particle in Stokes flow[5], where a BIM is used to solve for the velocity of spherical and ellipsoidal particles as a function particle size and position in the channel. Neutrally-buoyant, rigid spheres in Stokes flow are incapable of crossing streamlines, as can be argued by reversibility. In contrast, a deformable drop is capable of crossing streamlines due exclusively to deformation, such that the assessment of particle deformation and migration requires dynamical simulation.

THEORETICAL AND NUMERICAL METHOD

Consider an incompressible, Newtonian fluid of viscosity μ and density ρ, moving with a velocity **u**. The Stokes equations are valid at low Reynolds numbers (e.g. very slow fluid velocity, high fluid viscosity or minute flow domains). For the case of two immiscible fluids, the Stokes equations are subject to two boundary conditions: equality of interfacial velocities and continuity of the stress vector. The drops can be buoyant and the interfaces are assumed devoid of surfactants.

The fundamental solution for the Stokes velocity is given by the Green's function. The Green's function for the geometry of interest may be represented as sum of each contribution: $\mathbf{G}^k_{WALL} = \mathbf{G}^k_o + \mathbf{G}^k_{LW} + \mathbf{G}^k_{UW} + \mathbf{G}^k_{\varepsilon}$, for the free-space, lower-wall, upper-wall and numerical terms, respectively. The numerical term accounts for the presence of both walls. An advantageous property of this particular Green's function is that it does not require discretization of the walls or the surrounding fluid, but instead only discretization of the particle or drop surface. The form of the boundary-integral equation, made amenable for numerical calculations by deflation, is given by:

$$\mathbf{w}(\mathbf{y}) = \frac{2}{\lambda+1}\mathbf{u}^{\infty}(\mathbf{y}) + \kappa\left[2\int_S \mathbf{w}(\mathbf{x})\cdot\boldsymbol{\tau}^k(\mathbf{x};\mathbf{y})\cdot\mathbf{n}(\mathbf{x})dS_{\mathbf{x}} - \mathbf{w}'(\mathbf{y}) + \frac{\mathbf{n}(\mathbf{y})}{S}\int_S \mathbf{w}(\mathbf{x})\cdot\mathbf{n}(\mathbf{x})dS_{\mathbf{x}}\right] + \mathbf{F}(\mathbf{y}) \qquad (1)$$

$$\mathbf{F}(\mathbf{y}) = \frac{2}{(\lambda+1)}\left(\int_S [\frac{2k(\mathbf{x})}{\mathrm{Ca}} - \frac{\mathrm{Bo}}{\mathrm{Ca}}\mathbf{g}_o\cdot\mathbf{x}]\mathbf{G}^k(\mathbf{x};\mathbf{y})\cdot\mathbf{n}(\mathbf{x})dS_{\mathbf{x}} + \mathbf{u}^{\infty}(\mathbf{y})\right) \qquad \mathbf{y}\in S \qquad (2)$$

where S is the interface of the two immiscible fluids, the deflated velocity is **w(y) = u** - κ**u'**, where **u** is the physical velocity on S, $\lambda = \mu_{\mathrm{drop}}/\mu_{\mathrm{fluid}}$, $\kappa = (\lambda-1)/(\lambda+1)$, $\mathbf{u}_\infty$ is the undisturbed ambient fluid velocity, $k(\mathbf{x})$ is the mean curvature, γ is the surface tension, and $\boldsymbol{\tau}^k$ is the stress tensor corresponding to the fundamental solution $\mathbf{G}^k$. Two dimensionless quantities of physical interest are the capillary and Bond numbers. The capillary number, Ca = $\mu_{drop}U_c/\gamma$, gives the ratio of the viscous to interfacial forces, where U_c is the centerline velocity of the parabolic flow (used as the reference velocity). The Bond number, Bo = $\Delta\rho g_c H^2/\gamma$, where $\Delta\rho$ is the density difference between the two fluids, H is the channel height, and g_c is the magnitude of the gravitational force. Bo incorporates gravitational forces into (2) and is an indication of the ratio of body to interfacial forces. Following singularity subtraction, (1) becomes suitable for efficient numerical integration. Our algorithm involves discretization of the drop surface, followed by evaluation of (1) and a subsequent passive mesh stabilization technique, for every timestep. Discretization of the initially spherical drop is performed by division of the surface into a number of triangles (N_Δ), by inscription of either a dodecahedron or an icosaedron. Equation (1) is then solved at every triangle vertex. For $\lambda \neq 1$, the velocity is obtained by an iterative procedure using simple iterations. When $\lambda = 1$, only (2) requires calculation and the iterative procedure is not required.

RESULTS

When Ca→0, the drop shape does not change and the problem simplifies to the analysis of a spherical drop moving between two parallel walls. The drops cannot deform and therefore, cannot cross streamlines. Steady-state calculations for the velocity of spherical drops are given in Figure 1 for flow along the centerline at various λ. Simulation results for large λ agree well with previous results for rigid spheres[5]. The lateral migration of a single drop between two parallel walls has been examined at finite Reynolds number[6]. Figure 2 shows low-Reynolds number results for the case when the initially spherical drop is placed near the lower wall and allowed to deform and migrate. The drop reaches a steady-state shape and velocity in the center of the channel for the conditions shown.

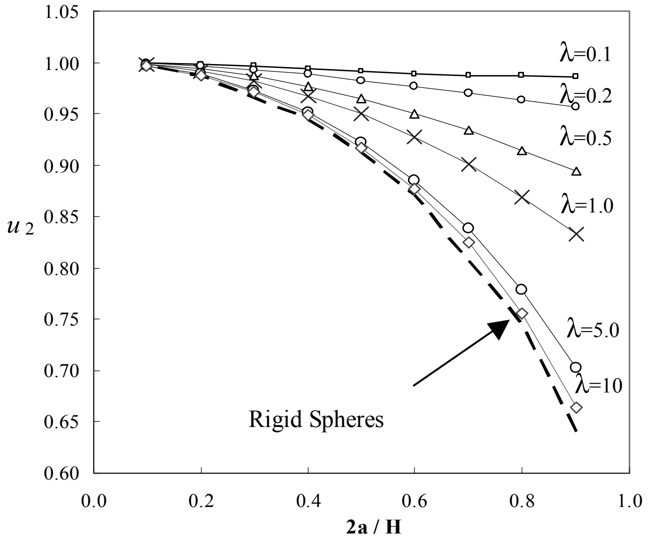

Figure 1. Translational velocities along the channel (u_2) of spherical drops for various viscosity ratios as a function of relative drop size, 2a/H, where a is the drop radius. The dashed line represents data for rigid spheres. The corresponding viscosity ratios are indicated above the curves.

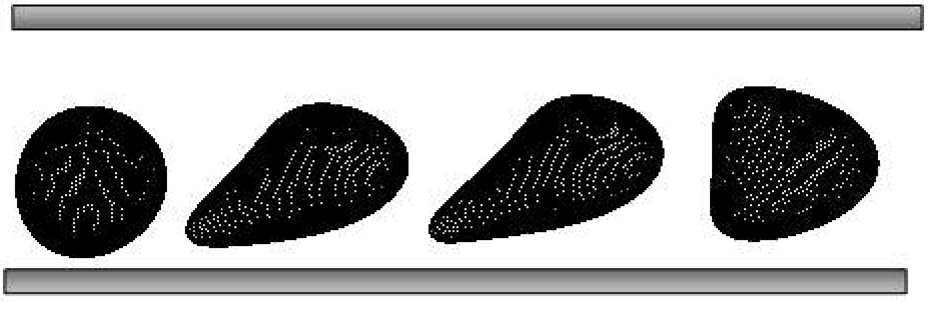

Figure 2. Evolution of a deforming drop initially placed near the lower wall. Migration of the drop across streamlines occurs. Here, Ca =1.0, λ = 1.0, Bo = 0, and N_Δ = 8640.

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