Convection driven by tidal heating: numerical model and parameterized theory

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1. Introduction

The role of solid state convection in the planet's interior is recognized for the Earth. The convection in the Earth's mantle is a driving force of tectonic and volcanic processes described by the plate tectonics theory (e.g. Czechowski, 1993a). There is no plate tectonics in the same form on the other planets and satellites but large-scale tectonic structures on other planetary bodies are probably a result of thermal convection in the bodies' interior (e.g. Schubert et al., 2001; McKinnon, 1998).

Thermal convection requires a source of heat. The main source of heat in the terrestrial planets is a decay of long-time living radioactive elements (U, Th and radioactive isotope of K). However, the radiogenic heating is not intensive enough to give rise convection in small bodies like medium size icy satellites (MIS) of giant planets. This group of bodies have radii in the range form 200km (Mimas, satellite of Saturn) to 780 km (Titania, satellite of Uranus). Czechowski and Leliwa-Kopystynski (2003) have found a strong correlation of the value of tidal parameters with the existence of large-scale tectonic structures on MIS. There are also others observational as well as theoretical indications that the tidal heating is the only possible source of heating that could lead to solid state convection in the interiors of MIS.

2. Numerical model and its basic results

To investigate the problem author developed 3D numerical model of convection driven by both: uniform radiogenic heating and non-uniform and non-spherically-symmetric tidal heating. The numerical model is based on the following system of equations: Navier-Stokes equation, equation of thermal conductivity, equation of continuity and equation of state. The Oberbeck-Boussinesq approximation of the equations is used (e.g. Czechowski, 1993 b), i.e. fluid is assumed to be incompressible and all parameters of the fluid (i.e. density, coefficient of thermal expansion, specific heat and thermal conductivity) are assumed to be constant with the exception of density in the gravity force term in the Navier-Stokes equation. For more details of the model see: Czechowski and Leliwa-Kopystyński, 2003) .

Rate of heat generation is the sum of two terms representing two different physical phenomena responsible for the heating: decay of radioactive elements and tidal deformations. The radiogenic heating is assumed to be constant. The tidal heating is not uniform and depends on the position inside the satellite. The distribution of tidal heat generation is calculated using numerical model developed by L. Czechowski. The model is based on the methods presented by Kaula (1963), Peale and Cassen (1978) and Poirier et al. (1983). Tensor of tidal deformations in a satellite is expressed by truncated series of spherical functions. The rheology of the satellite interior is assumed to be of Kelvin-Voigt type and the eccentricity of the orbit is included as the only source of deformations. The adopted approximation makes possible to scale the distribution of the tidal heating. It means that the same distribution after multiplying by a constant coefficient could be used for different values of eccentricity.

Steady state solutions of the convective flow are obtained using the numerical model for low and moderate Rayleigh number ($0<Ra<500\,000$). The results indicate that convection patterns specific for the tidal heating are strongly oriented in respect to the near and the far sides of the satellite. The pattern of convection consists usually of 2 cells, but for some cases 1 cell pattern is also observed (for more details of the results see: Czechowski and Leliwa-Kopystyński, 2003) .

3. Function Nu(Ra)

The Rayleigh number is the main parameter that could serve as a measure of intensity of convection. The role of convection for heat transfer is determined by Nusselt number Nu. For considered here problem of steady state convection in volumetrically heated body, the Nusselt number could be defined as: Nu=DTcond/DTconv
where DTcond is the average temperature difference in the body without convection (the surface temperature is chosen to be a reference temperature) and DTconv is the average temperature difference in the body with convection. If convection is absent then thermal regime is determined by conduction only and Nusselt number is equal to 1. If convection operates in the body, the heat transfer is more efficient comparing to the heat transfer without convection, DTconv is lower than DTcond and consequently the values of Nu greater than 1 are characteristic for convection. Generally the Nusselt number is an increasing function of the Rayleigh number

but scrutiny of the function Nu(Ra) indicates also some exceptions from the above rule, especially close to bifurcations.

The function Nu(Ra) is usually approximated with some kind of the power function. For considered here case the situation is a little more complicated because the critical value of the Rayleigh number is zero, but power function is still useful with some modifications. Eventually the following approximation is chosen: Nu(Ra)=(Ra-A)^M where A and M are determined by fitting data obtained from the numerical model. Both values: A and M depend on ratio of tidal to radiogenic heating.

4. Parameterized theory

Parameterized theory of convection is used successfully for convection driven by radiogenic heating. The basic assumptions of this method is presented in many papers: e.g. Schubert et al. 2001. Some modification of the theory is necessary for convection driven by tidal heating. In the present research the parameterized theory is based on the following system of equations: function Nu(Ra) (see discussion above), viscosity of water ice as a function of temperature (taken from McKinnon, 1998), efficiency of tidal heating (according to the results of Poirier et al. 1983), thermal conductivity and specific heat of ice as a function of temperature (both taken from Leliwa-Kopystyński and Kossacki, 2000). The main unknown of the system is average temperature of the body's interior.

5. Discussion

The main result of the presented parameterized theory is determining the existence of a few steady state solutions for convection with significantly different energies. Each of the satellites has a low temperature solution that corresponds to low tidal heating and slow convective flow. A given satellite could be in any state, but beside Enceladus all other Saturnian medium sized icy satellites seem to be in the low energy state.

The answer why Enceladus is in the high energy state is beyond the scope of the present paper. Large impact in the past could be a source of additional energy. This energy could increased the temperature and consequently the efficiency of tidal heating. The absence of signs of such catastrophic event makes this hypotheses doubtful. Therefore the author prefers rather that the present state of Enceladus is an effect of specific evolution of the satellite's orbit.

6. Conclusions

1. The parameterized theory indicates the existence of a few steady states of convection with significantly different energies for some satellites.
2. Enceladus is in high energy state. All other medium sized Saturnian icy satellites seem to be in the basic, low energy state.
3. The average rate of tidal heat production for basic state is comparable to the radiogenic rate for a low energy state. For a high energy state the rate of tidal heating is 10 to 100 higher than radiogenic heating.
4. The Rayleigh number is so sensitive function of parameters that its reliable estimation cannot be achieved by the parameterized theory. Theory could be used for determining some less sensitive rheological parameters.

7. References

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