XXI ICTAM, 15-21 August 2004, Warsaw, Poland

EXPERIMENTAL ANALYSIS OF MODAL INTERACTIONS IN THE NON-LINEAR VIBRATIONS OF A PLATE

Pedro Ribeiro

IDMEC/DEMEGI, Fac. de Engenharia, Univ. do Porto, Rua Dr. Roberto Frias, 4200-465 Porto, Portugal

Summary The geometrically non-linear vibrations of a fully clamped plate are experimentally investigated. The plate is excited transversely with harmonic excitations and the amplitudes of the first and higher harmonics of the response are analysed at different points. It is verified that internal resonances occur between the first and higher order modes, that is, that due to the non-linearity of the system, energy is transferred from the first to higher order modes.

INTRODUCTION

Under the action of harmonic excitations, geometrically non-linear plates often experiment periodic, but non-harmonic, vibrations. In this case, the natural frequencies and the mode shapes of vibration usually change with the vibration amplitude, because the stiffness of the system is not constant [1-3]. Therefore, the natural frequencies can easily become commensurable, that is, related by an equation of the form $m_1\omega_1+m_2\omega_2+ \ldots + m_n\omega_n=0$, where m_i are integers. When this happens, and due to the coupling caused by the non-linearity, energy may be interchanged between the different modes of vibration. As a result of this internal resonance phenomenon [2] the structure responds in the two or more coupled modes. Although, the strong effects that modal coupling have on the non-linear response have been predicted in numerical analysis of fully clamped plates [3] and in the experimental analysis of beams [4], there seems to lack in the literature an experimental verification of this interesting phenomenon that includes the actual definition of the mode shapes,.

In this work, the geometrically non-linear vibrations of a fully clamped aluminium plate are experimentally investigated. The main goals are to verify how important the non-fundamental harmonics can be, where fundamental indicates the frequency of excitation and to search for internal resonances. Moderate vibration amplitudes are considered.

EXPERIMENTAL ANALYSIS

Procedure

The dimensions of the aluminium alloy plate analysed are 235×220×1 mm. Approximately fully clamped conditions are implemented by a rigid and heavy fixture, whose natural frequencies are quite higher then the ones of the plate. A grid of 7×7 measuring points was marked on the plate and an electromagnetic exciter was connected to it by a drive rod and a force transducer. In spite of its mass, it was decided to always use the force transducer, because it is necessary to adjust the gain of the amplifying system in order to keep the amplitude of excitation constant for different frequencies. Moreover, the force transducer – often with the help of an accelerometer – allows one to verify that the signal applied in the studies of non-linear vibrations is sinusoidal. The accelerations are measured using two very light weight accelerometers.

The signal sent to the exciter is generated by an analyser, constituted by software, a PC and a front-end. The linear natural frequencies are determined from frequency response functions, after applying low amplitude random excitations. In the non-linear studies, the plate is excited with sinusoidal excitations at frequencies close to its first natural frequency and, at each excitation amplitude, the frequency of excitation is slowly changed upwards and downwards.

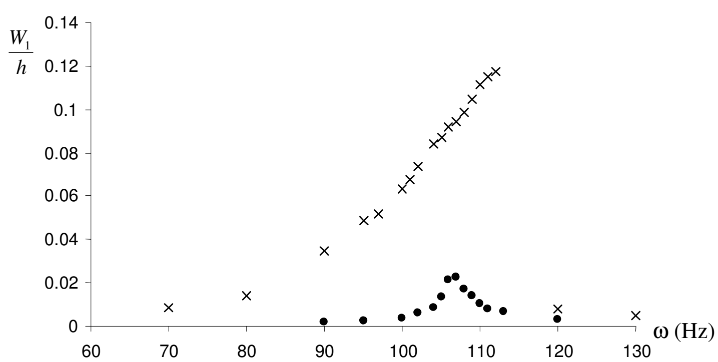

Figure 1. Normalized displacement amplitude, 1º harmonic, excitations with amplitudes: ? 100 mN, × 400 mN.

Resonance curves and coupled modes

Several amplitudes of excitation will be applied, but, in this extended abstract, only the amplitudes of the vibration displacements of the plate's central point for excitations with 0.1 N and 0.4 N are shown (Figure 1). The resonance curve bends towards the right as the amplitude of vibration increases, showing the expected hardening spring behaviour.

Figure 2 represents the autospectra of the accelerations of the central point, when the excitation is harmonic with 113 Hz frequency and two amplitudes: 0.1 N and 0.4 N. One verifies that, due to the non-linearity, the third harmonic is very important at the larger vibration amplitude and negligible at the lower one.

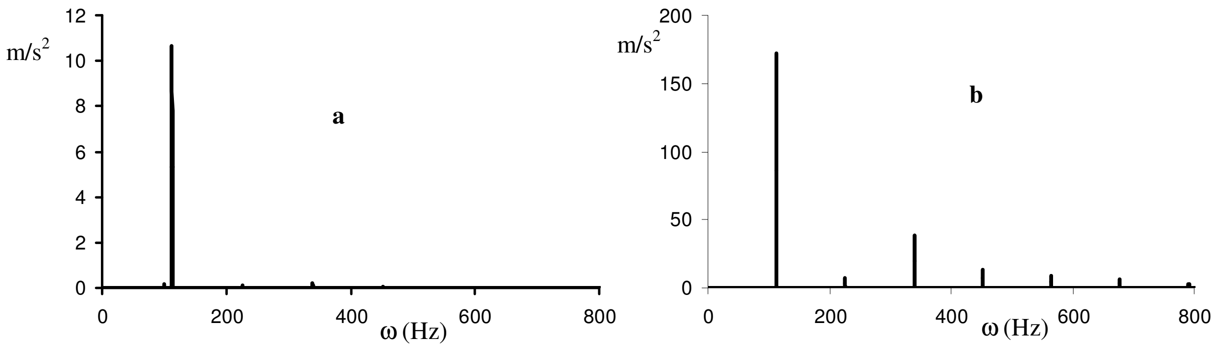

Figure 2. Autospectra of acceleration due to excitations with 113 Hz and (a) 0.1 N, (b) 0.4 N.

The excitation of the third harmonic is particularly strong when the first and fourth modes (Figure 3), whose linear frequencies are related by a factor close to 3, become coupled due to a 1:3 internal resonance. This will be demonstrated by defining the deformations associated with each of the excited harmonics, which form the non-linear mode shapes.

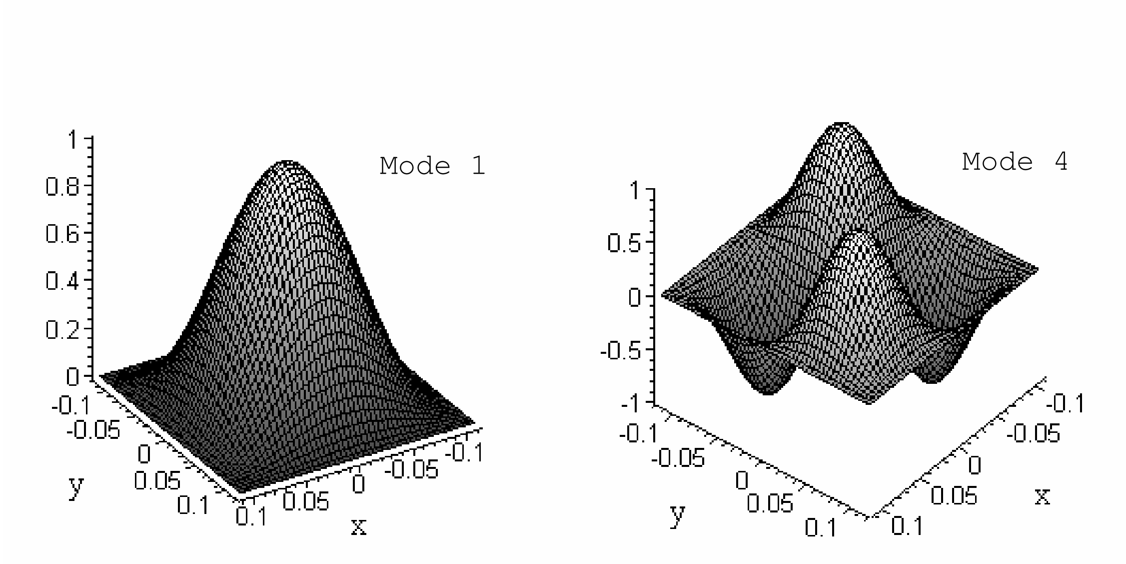

Figure 3. Plate's first and fourth linear modes of vibration.

ACKNOWLEDGMENTS

The support of this work by the Portuguese Science and Technology Foundation, through project POCTI/1999/EME/32641, FEDER, is gratefully acknowledged.

References

[1] Nayfeh A. H., Balachandran, B.: Modal interactions in dynamical and structural systems, *Applied Mechanics Review* **42** S175 - S201, 1989.

[2] Szemplinska-Stupnicka W.: The behaviour of nonlinear vibrating systems. Kluwer Academic Publishers, Dordrecht 1990.

[3] Ribeiro P., Petyt M.: Non-linear free vibration of isotropic plates with internal resonance. *Int. J. of Non-linear Mechanics* **35** 263-278, 2000.

[4] Ribeiro P., Carneiro R.: Experimental detection of modal interaction in the non-linear vibration of a hinged-hinged beam. *J. of Sound and Vibration* **277** 943-954, 2004.