LEVEL-SETS AND MIXED APPROACHES FOR DYNAMIC CONTACT PROBLEMS

Hachmi Ben Dhia*, Chokri Zammali*

*Ecole Centrale Paris, Labo. MSS-Mat, 92295 Châtenay-Malabry, France

Summary The Signorini-Moreau contact conditions are set as equations by using unknown *Level-Set* fields. From this, a mixed weak-strong formulation of dynamic contact problems is derived; the *Level-Set* fields being the intrinsic contact unknown fields. The problem is then discretized by time, space and collocation schemes and some numerical experimentations are carried out, showing the effectiveness of our global approach. The paper is ended with a prospective use of the local multiscale *Arlequin* method [1].

INTRODUCTION

Impact problems are nonlinear in essence, but also irregular and multiscales in space and time. Theses evidences are shown by analytical investigations giving explicit solutions of simple model impact problems (see e.g. Gloldsmith [2]). General industrial impact problems have however to be approximated by means of numerical tools. Hughes et al. [3], have designed special numerical schemes since the late seventies and increasing efforts have been since furnished by the computational contact community to address this numerical issue (see e.g. [4] and the sampling references therein). Very important advances have this way been done and some of the most pertinent schemes are now capitalized in commercial codes. But to our best knowledge and due to the aforementioned intrinsic difficulties of the impact problems (strong nonlinearities and both space and time irregularities and multiscale intrinsic characters), none of the available numerical schemes is yet sufficiently robust, efficient and accurate to address for instance dynamic contact problems such as those involved by vibrating structures under impact. The present work is a contribution to this wide and complex theme.

Using the "*viability lemma*" of J.J. Moreau [5], the unilateral contact laws are written in a first part as equations by using two unknown possibly multivoque *Level-Set* type fields standing for i) a location of the position of contact surfaces with respect to each other and ii) the *Signe* of the normal velocity jump on the interface. A weak-strong formulation of dynamic unilateral contact conditions is then derived in a straightforward manner. In a second part, an original continuous weak-strong mixed formulation of a dynamic 3-D contact problem is carried out. The associated discrete nonlinear systems are then derived by using a θ-scheme time discretization, a Galerkin method and a collocation one. The numerical solution strategy used to solve this problem is briefly commented and numerical tests are carried out in a third part. It is particularly shown that the numerical solutions do not exhibit pathologies (due to shocks) such as spurious oscillations of the numerical mechanical fields. The paper is ended with a prospective use of the local multiscale *Arlequin* method [1] to superpose to the global (coarse) model local refined ones just in the neighbourhood of impacted zones in order to save human and machine resources. First promising numerical results exemplifying this approach are given.

A DYNAMIC CONTACT MODEL

Let us consider two solids (elastic, for simplicity) which may come into (frictionless, for clarity) contact/impact during their evolution. The classical Signorini contact laws can then be set as flollowing (with rather classical notations) :

$$d_n \leq 0,\ \ \lambda \leq 0\ \text{ and }\ \lambda\, d_n = 0,\ \text{ on }\ \Gamma_c \times I \tag{1}$$

where Γ_c, I, d_n and λ are the potential "slave" contact surface, the time interval, the so-called signed distance and the normal contact pression, respectively.

By using the "viability lemma" of J.J. Moreau [5], and by assuming that at the initial time, the conditions (1) are satisfied, these conditions can then be written in an equivalent manner in terms of placements and velocities as follows.

If $d_n < 0$ then a free boundary condition is imposed

$[[v_n]] \leq 0$, $\lambda_v \leq 0$ and $\lambda_v\,[[v_n]] = 0$, otherwise, with $[[v_n]]$ standing for the normal velocities-jump

By using the *Signe* function introduced in [6], these Signorini-Moreau inequalities can be transformed to equalities:

$$\lambda_v = \mathbf{S}_d \mathbf{S}_v \left(\lambda_v - h[[v_n]]\right),\ \text{on}\ \Gamma_c \times I \tag{2}$$

$$\mathbf{S}_d = 1_{\mathrm{IR}^-}\left(-d_n\right),\ \mathbf{S}_v = 1_{\mathrm{IR}^-}\left(\lambda_v - \rho[[v_n]]\right),\ \text{on}\ \Gamma_c \times I \tag{3}$$

with $h \neq 0$, $\rho > 0$ and 1_{IR^-} the characteristic function of the semi-negative axis.

This new setting of Signorini-Moreau contact conditions is interesting from a numerical point of view since it leads itself "naturally" to a weak formulation, well-suited to numerical approximations.

WEAK-STRONG FORMULATIONS OF THE DYNAMIC CONTACT PROBLEM

By using the Virtual Power Principle *VPP* (in which a behaviour law is used to express the First Piola-Khirchhoff stress $\mathbf{\Pi}^i$) and a weak formulation of equation (2), keeping equations (3) as local strong ones, the following (original, to our knowledge) weak-strong mixed formulation of the dynamic contact problem is obtained:

Find ($\mathbf{u}^i$, $\mathbf{v}^i$, λ_v, $\mathbf{S}_d$, $\mathbf{S}_v$) such as for all admissible $\mathbf{w}^i$ and λ^* and for t in I,

$$\left\{\begin{array}{l} \sum_{i=1}^{2}\int_{\Omega_0^i}\rho_0^i\dot{\mathbf{v}}^i.\mathbf{w}^i d\Omega+\sum_{i=1}^{2}\int_{\Omega_0^i}\mathbf{\Pi}^i:\nabla(\mathbf{w}^i)d\Omega-\int_{\Gamma_c}\mathbf{S}_d\mathbf{S}_v\lambda_v[[w_n]]d\Gamma=0 \\ -\frac{1}{h}\int_{\Gamma_c}\left[\lambda_v-\mathbf{S}_d\mathbf{S}_v\left(\lambda_v-h[[v_n]]\right)\right]\lambda^* d\Gamma=0 \end{array}\right. \quad \left\| \quad \begin{array}{l} \mathbf{u}^i(t)=\mathbf{u}^i(t_0)+\int_{t_0}^{t}\mathbf{v}^i(\tau)d\tau \\ \mathbf{S}_d=1_{\mathrm{IR}^-}\left(-d_n\right),\quad \mathbf{S}_v=1_{\mathrm{IR}^-}\left(\lambda_v-\rho[[v_n]]\right) \text{ on } \Gamma_c \end{array}\right. \qquad (4)$$

where $\mathbf{u}^i$, $\mathbf{v}^i$ stand for the displacement and velocity fields.

We notice that since the velocity fields may be locally discontinuous with respect to time, the time derivative in the *VPP* is to be understood in an appropriate sense. We notice also that although the above formulation may recall available lagrangian formulations of frictionless contact problems, it is original to our best knowledge: it is a non augmented, non constrained lagrangian formulation of dynamic contact problems.
To be solved, this problem is discretized by means of three numerical tools. A low-order finite time difference scheme of Euler type is used to approximate partial time derivatives. A Galerkin space discretisation method (of the finite element type, or spectral one) is used to approximate the continuous fields $\mathbf{u}^i$, $\mathbf{v}^i$ and λ_v, *w*hile respecting the compatibility condition (see [7], in the quasistatic case). The *Level-Set* like fields $\mathbf{S}_d$, $\mathbf{S}_v$ are approximated by means of a collocation method, related intimately in practice to numerical contact integration points. The nonlinear discrete systems will be detailed during the conference.

NUMERICAL SOLUTION STRATEGY AND RESULTS

Our non linear systems are solved, at each time step, by a fixed point strategy on the *Level-Set* fields $\mathbf{S}_d$ and $\mathbf{S}_v$ (actually on their values on the selected finite collocation contact points, for the discrete systems), coupled with classical numerical methods for the solution of the others nonlinearities of the problem.
Let us now give a numerical illustration. We consider the very well-known shock of two bars of identical mechanical characteristics and represent the interface mechanical fields obtained by using: i) the Signorini contact conditions (1) and a Newmark scheme, with numerical damping and ii) the suggested approach (figure 1). Other results showing the effectiveness of our global approach of dynamic contact problems, (taking also into account friction phenomena), will be given during the conference.

A PROSPECTIVE USE OF THE ARLEQUIN FRAMEWORK

Our work is ended by showing the use of the local multiscale Arlequin method [1] to introduce, with great flexibility while saving machine resources, a mechanical and numerical refinement of impact problems in critical zones. The Taylor bar test, treated in the Arlequin framework by superposing a local elastoplastic refined model to a global elastic one, is shown in figure 2.

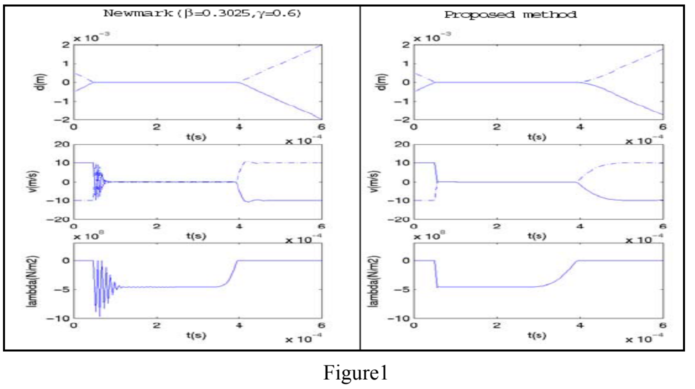

Figure1

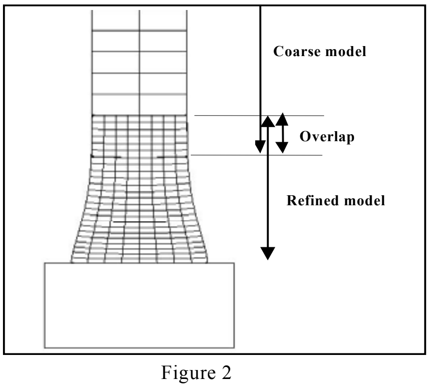

Figure 2

References

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