XXI ICTAM, 15–21 August 2004, Warsaw, Poland

A NOVEL CONTACT MODEL BASED ON VOLUMETRIC INFORMATION

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Summary A novel contact model is presented which includes normal contact force and damping, rolling resistance torque and tangential friction force. It features a contact stiffness proportional to the contact area and leads automatically to the correct selection of the point of action of the force, which is shown to be proportional to the interpenetration volume. A numerical simulation of a sphere impacting on the inside surface of a cylinder is presented.

INTRODUCTION

Contact models often drastically simplify the interaction of complexly shaped bodies to simple cases, or even point contacts. The significance of the shapes of the colliding bodies on the resulting contact forces is generally not taken into account, or not modeled in a consistent manner for all components of the contact force, namely, normal and tangential force, rolling and spinning resistance torques.

The Hertzian contact model, which assumes an elastic half-space approximation, is often used to model contact phenomena. However, this approximation is not valid for bodies that have conforming surfaces. Strictly speaking, the Hertz theory should be used to model the contact mechanics only when the colliding body surfaces can be described by second order polynomials. However, in practice its application can be extended to bodies with smooth surfaces as long as the resulting contact area remains small with respect to the dimensions of the bodies. Even though the geometric boundary conditions may not be exactly fulfilled, the Hertz theory's assumption that the pressure distribution over the contact area is elliptical will still apply, or at least provide a good approximation, such that the theory still applies, but only for non-conforming geometries.

Johnson [1] suggests to use the Winkler elastic foundation as a simple approximation in complex situations where half-space theory would be very cumbersome. Hippmann [2] uses this approach to deal with complex, i.e., conforming, polygonal, geometries. The contact force is derived by numerically summing up the contribution of each polygon using a simple contact model.

In this paper, a model for distribution the pressure over the contact area will be presented. Using this model, it is shown how a complete contact model can be analytically derived by integrating the contact pressure over the area. This novel approach includes all forces and moments resulting from the contact mechanics effects between the colliding bodies, and in particular it includes the rolling resistance torque.

NORMAL AND TANGENTIAL FORCE MODEL

The Winkler elastic foundation model (Figure 1) predicts that the pressure $p(\underline{s})$ at any location $\underline{s}$ on the contact surface S is proportional to $h_s(\underline{s})$, the interpenetration depth between the two undeformed surfaces at point $\underline{s}$. However, it does not include any damping effect to dissipate energy during the contact process. The energy dissipation is best modeled using a hysteretic damping term a [3]. Therefore, a new pressure distribution model is proposed as

$$p(\underline{s}) = \frac{df_n(\underline{s})}{dS} = \frac{k_f}{h_f} h_s(\underline{s}) \left(1 + a\, v_n(\underline{s})\right) , \tag{1}$$

where k_f is the elastic modulus of the foundation, h_f is the depth of the foundation mattress, and v_n is the relative velocity component normal to S at $\underline{s}$. The volume V spanned by the intersection of the two undeformed geometries of the colliding bodies is labeled the *volume of interference*. By definition, the volume of interference contains the S. The volume of interference and its centroid $\underline{r}_c$ are given by

$$V = \int_S h_s(\underline{s})\, dS = \int_V dV \; ; \; \underline{r}_c = \frac{1}{V}\int_V \underline{r}\, dV \; , \tag{2}$$

where $\underline{r}$ is a point in the volume of interference. Integrating $df_n(\underline{s})$ over S and using Equations (2) the contact force results as

$$\underline{f}_n = \frac{k_f}{h_f} V \left(1 + a\, v_{cn}\right) \underline{n} \; , \tag{3}$$

where v_{cn} is the normal component of the relative velocity at $\underline{r}_c$. The tangential friction is computed as given in [3] using this new definition of $\underline{f}_n$ as given in Equation (3).

The interference volume, its centroid, and the contact surface normal, necessary to evaluate Equation (3), can be derived either analytically or numerically from geometric and kinematic information. This model features a contact stiffness proportional to the contact area and defines the location of the point of action of the contact force as $\underline{r}_c$. Contact models based on penetration depth typically assume the point of action to be located at the point of maximum penetration. However this causes problems when the contact area is large and the surfaces of the colliding bodies are nearly parallel, i.e. instantenous change of point of action and of the moment arm of the contact force w.r.t. the colliding bodies.

ROLLING FRICTION MODEL

The shape of load distribution across the contact area gets distorted due to the presence of the hysteretic damping term. Hence, the centroid of the load distribution is not at the same location as the centroid of the interpenetration volume. If the point of application is selected as $\underline{r}_c$, an additional rolling resistance torque has to be applied.

The torque about $\underline{r}_c$ resulting from the contact force distribution over area of contact is computed by integrating the infinitesimal rolling friction torque $d\underline{\tau}_r(\underline{s}) = \underline{\rho}_s \times d\underline{f}_n(\underline{s})$ over S, where $\underline{\rho}_s = \underline{s} - \underline{r}_c$. It can be shown that

$$\underline{\tau}_r = \frac{k_f}{h_f} a\, \mathbf{J}_c \cdot \underline{\omega}_t \; ; \;\; \mathbf{J}_c = \int_S \left((\underline{\rho}_s \cdot \underline{\rho}_s)\, \mathbf{I} - \underline{\rho}_s \underline{\rho}_s \right) h_s(\underline{s})\, dS \;\; , \tag{4}$$

where $\underline{\omega}_t$ is the component of the relative angular velocity tangential to S. Since the interpenetration depth is typically small, $\mathbf{J}_c$ is closely approximated as the second moment of volume about the centroid of V.

When the bodies are rolling with respect to each other, then $\underline{\tau}_r$ is the resistance torque acting to prevent the rolling. Equation (4) shows that if the two bodies are not rolling with respect to each other ($\underline{\omega}_t = 0$), then the normal force is distributed symmetrically around the volume of interference centroid, so the net moment at the centroid is zero.

NUMERICAL EXAMPLE: BOUNCING SPHERE

The model described above was utilized to simulate a sphere bouncing inside a cylinder. Figure 2 shows the three-dimensional trajectory of the sphere center. Figure 3 shows the decays in both of the angular and translational velocities over time. Observe that the sphere eventually stops moving in y-direction. Clearly, the resistance torques act to prevent rotational motion, as we expected. The parameters used are give in [3].

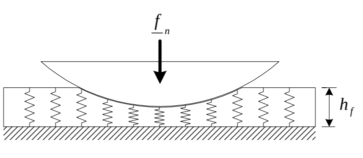

Figure 1. The Winkler elastic foundation model.

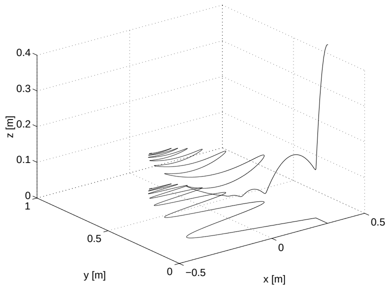

Figure 2. Trajectory of sphere center.

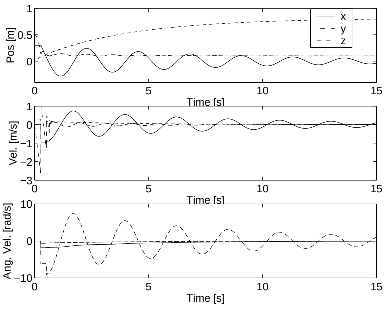

Figure 3. Position and velocity components versus time.

CONCLUSIONS

In this paper, a novel contact model was presented which includes normal contact force and damping, rolling resistance torque and tangential friction force. A modified Winkler elastic foundation model was introduced to obtain the pressure distribution across the contact area. The contact force is expressed analytically by integrating the pressure distribution over the contact area. The resulting model features a contact stiffness proportional to the contact area and leads automatically to the correct selection of the point of action of the force, which is shown to be proportional to the interpenetration volume.

The proposed model is not restricted to contact situations where the bodies have non-conforming geometries, but can be used for any geometry with reasonably flat contact area. A numerical simulation example of a sphere colliding on the interior surface of a cylinder under the action of gravity was provided. The resulting dynamic behavior is physically realistic and satisfies the condition $|\underline{v}| = R|\underline{\omega}|$ and it is seen that the effect of the resistance torque is to dissipate rotational energy such that the sphere's motion is eventually stopped.

References

[1] K. L. Johnson, *Contact Mechanics*. London: Cambridge University Press, 1985.

[2] G. Hippmann, "An algorithm for compliant contact between complexly shaped surfaces in multibody dynamics," in *ECCOMAS Multibody Dynamics Conference*, Lisbon, Portugal, July 1–4 2003.

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