LINEAR WAVES AND BAROCLINIC INSTABILITY IN AN INHOMOGENEOUS-DENSITY LAYERED PRIMITIVE-EQUATION OCEAN MODEL

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Summary We consider a multilayer generalization of Ripa's inhomogeneous-density single-layer primitive-equation model. In addition to vary arbitrarily in horizontal position and time, the horizontal velocity and buoyancy fields are allowed to vary linearly with depth within each layer of the model. Preliminary results on linear waves and baroclinic instability suggest that a configuration involving a few layers may set the basis for a quite accurate and numerically efficient ocean model.

INTRODUCTION

Because of the physical and practical relevance of layered models, methods to accommodate thermodynamic processes into these models have been developed. The simplest method consists of allowing the buoyancy field to vary in horizontal position and time, but keeping all dynamical fields as depth independent. This is formally achieved by replacing the horizontal pressure gradient by its vertical average [4]. The resulting inhomogeneous-density layered models, usually referred to as “slab” models, have been extensively used in ocean modeling.

Despite their widespread use, slab models are known to have several limitations and deficiencies [6]. For instance, a slab single-layer model cannot represent explicitly the thermal-wind balance which dominates at low frequency. This balance is fundamental in processes like cyclogenesis, which has been shown [3] to be incorrectly described with a slab single-layer model.

To cure the slab models limitations and deficiencies, Ripa [5] proposed an improved closure to partially incorporate thermodynamic processes in a one-layer model. In addition to allowing arbitrary velocity and buoyancy variations in horizontal position and time, in Ripa's model the horizontal velocity and buoyancy fields are allowed to vary linearly with depth. The model, which has been generalized to an arbitrary number of layers by Beron-Vera [1], enjoys a number of properties which make it very promising for applications.

As a base test for the validity of the generalized Ripa's model we consider linear waves, focusing particularly on vertical normal-mode phase speeds, and classical baroclinic instability.

THE GENERALIZED MODEL

The starting point to generalize Ripa's single-layer model is to consider a stack of n active fluid layers of thickness $h_i(\mathbf{x}, t)$. These layers can be limited from below by an irregular, rigid surface and from above by a soft surface or vice versa. We call the former possibility a rigid bottom setting and the latter a rigid lid setting. Above (resp., below) the active fluid there is an inert, infinitely thick layer of lighter (resp., denser) fluid in the rigid bottom (resp., lid) case. For the ith-layer horizontal velocity and buoyancy relative to the inert layer, respectively, one must then write

$$\mathbf{u}_i(\mathbf{x}, \sigma, t) = \bar{\mathbf{u}}_i(\mathbf{x}, t) + \sigma \mathbf{u}_i^\sigma(\mathbf{x}, t), \quad \vartheta_i(\mathbf{x}, \sigma, t) = \bar{\vartheta}_i(\mathbf{x}, t) + \sigma \vartheta_i^\sigma(\mathbf{x}, t). \tag{1}$$

Here, the overbar stands for vertical average within the ith layer, and σ is a scaled vertical coordinate which varies linearly from ± 1 at the base of the ith layer to ∓ 1 at the top of the ith layer. [The upper (resp., lower) sign corresponds to the rigid bottom (resp., lid) configuration.] The generalized model equations, which follow upon replacing (1) in the continuously and arbitrarily stratified (i.e. exact) primitive equations, namely rotating incompressible hydrostatic Euler–Boussinesq equations, then take the form

$$\partial_t h_i + \boldsymbol{\nabla} \cdot h_i \bar{\mathbf{u}}_i = 0, \tag{2a}$$

$$\overline{\mathrm{D}_i \vartheta_i} = 0, \quad (\mathrm{D}_i \vartheta_i)^\sigma = 0, \tag{2b,c}$$

$$\overline{\mathrm{D}_i \mathbf{u}_i} + f\hat{\mathbf{z}} \times \bar{\mathbf{u}}_i + \overline{\boldsymbol{\nabla} p}_i = \mathbf{0}, \quad (\mathrm{D}_i \mathbf{u}_i)^\sigma + f\hat{\mathbf{z}} \times \mathbf{u}_i^\sigma + (\boldsymbol{\nabla} p_i)^\sigma = \mathbf{0}. \tag{2d,e}$$

Here, f is the Coriolis parameter (twice the local angular rotation frequency); $\hat{\mathbf{z}}$ is the vertical unit vector; $\boldsymbol{\nabla}$ is the horizontal gradient; $\overline{\mathrm{D}_i \bullet}$ and $(\mathrm{D}_i \bullet)^\sigma$ are, respectively, the vertical average and σ components of the ith-layer material derivative; and $\overline{\boldsymbol{\nabla} p}_i$ and $(\boldsymbol{\nabla} p_i)^\sigma$ are, respectively, the vertical average and σ components of the ith-layer pressure gradient force. [For details cf. Refs. 5,1.]

Some of the attractive properties of this model are the following. First, the model can represent explicitly within each layer the thermal-wind balance which dominates at low frequency. Second, volume, mass, buoyancy variance, energy, and momentum are preserved by the dynamics. Third, because of the possibility of vertical stratification within each layer, thermodynamic processes (e.g. heat and freshwater inputs across the ocean surface, vertical mixing, etc.) can be incorporated more realistically than in slab models.

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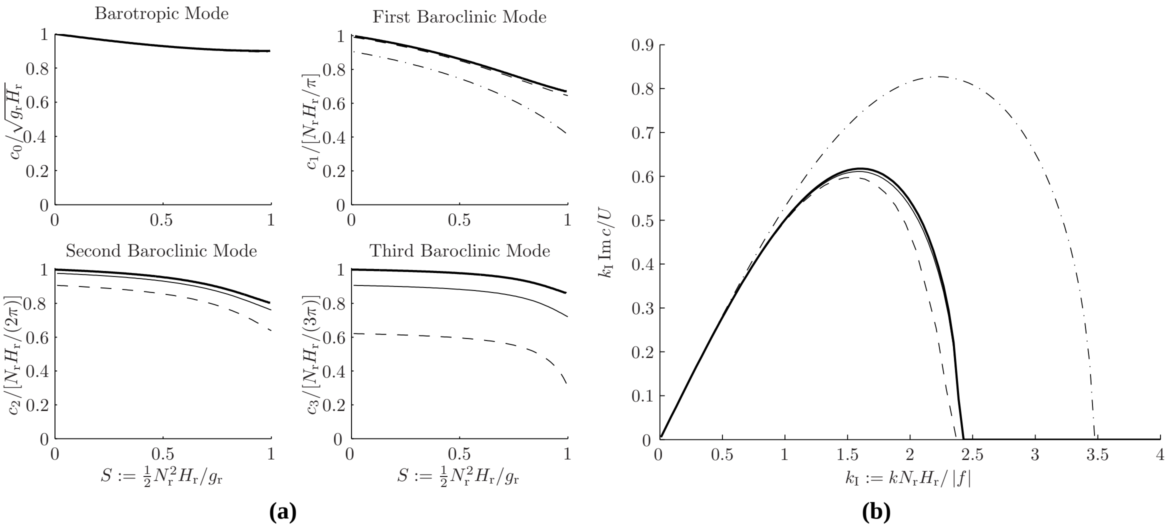

Figure 1. (a) Phase speed of long gravity waves as a function of the stratification in a reference state with no currents. (b) Growth rate of the most unstable Eady wave as a function of wavenumber. In both panels the exact result is indicated by a heavy-solid line, and the layered model predictions with dot-dashed (one layer), dashed (two layers), and light-solid (three layers) lines.

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System (2), linearized with respect to a *reference state* with no currents, can be shown to sustain the usual midlatitude and equatorial gravity and vortical waves in several vertical normal modes. In this work we concentrate on how well these modes are represented by considering the phase speed of long gravity waves in a reference state characterized by the stratification parameter $S := \frac{1}{2}N_\mathrm{r}^2H_\mathrm{r}/g_\mathrm{r}$. Here, N_r is the Brunt–Väisälä frequency, H_r is the total thickness of the active fluid layer, and g_r denotes the vertically averaged buoyancy. All these three quantities are held constant. The reference buoyancy then varies linearly from $g_\mathrm{r}(1 \mp S)$ at the top of the active layer to $g_\mathrm{r}(1 \pm S)$ at the base of the active layer. Physically acceptable values of S must be such that $0 < S < 1$ [5, 2]. Figure 1a compares, as a function of S, the phase speed of long gravity waves assuming exact dynamics and (2) with $n = 1$, 2, and 3. The comparison is perfect for the barotropic mode phase speed even including only one layer. The use of two layers amounts to an excellent representation of the first baroclinic mode phase speed, and a very good representation of the second baroclinic mode phase speed. To reasonably represent also the third baroclinic mode phase speed, no more than three layers are needed.

We now turn our attention to classical baroclinic instability. We thus consider a *basic state* with a parallel current in an infinite channel on the f plane, which has a uniform vertical shear and that is in thermal-wind balance with the across-channel buoyancy gradient. We further set the basic velocity to vary (linearly) from $2U$ at the top of the active layer to 0 at the base of the active layer. Accordingly, the basic buoyancy field varies from $g_\mathrm{r}(1 - 2fUy/H_\mathrm{r} \mp S)$ at the top of the active layer to $g_\mathrm{r}(1 - 2fUy/H_\mathrm{r} \pm S)$ at the base of the active layer (y denotes the accross-channel coordinate). Figure 1b compares in the classical Eady limit, as a function of the along-channel wavenumber the growth rate) of the most unstable normal-mode perturbation assuming exact dynamics and (2) with $n = 1$, 2, and 3. The comparison is almost perfect when three layers are included and very good when two layers are included. When only one layer is considered, the comparison is not as good but a high wavenumber cutoff of baroclinic instability is present. This important dynamical feature cannot be represented with a slab single-layer model.

PRELIMINARY CONCLUSIONS

We have tested the performance of a novel inhomogeneous-density primitive-equation layered model, which features vertical shear and stratification within each layer, in two important aspects of ocean dynamics, namely linear waves and baroclinic instability. Preliminary results suggest that a model with a small number of layers may be used as a basis for a quite accurate and numerically economic ocean model. To make stronger statements on the model's accuracy and efficiency, fully nonlinear, forced–dissipative problems must of course be considered. This work is currently underway.

References

[1] Beron-Vera, F. J.: Multilayer primitive-equation ocean model with variable vertical shear and stratification within each layer. *J. Fluid Mech.*, to be resubmitted, 2004 (e-Print arXiv:physics/0312083).
[2] Beron-Vera, F. J. and P. Ripa: Free boundary effects on baroclinic instability. *J. Fluid Mech.* **352**:245–264, 1997.
[3] Eldevik, T.: On frontal dynamics in two model oceans. *J. Phys. Oceanogr.* **32**:2,915–2,925, 2002.
[4] Ripa, P.: Conservation laws for primitive equations models with inhomogeneous layers. *Geophys. Astrophys. Fluid Dyn.* **70**:85–111, 1993.
[5] Ripa, P.: On improving a one-layer ocean model with thermodynamics. *J. Fluid Mech.* **303**:169–201, 1995.
[6] Ripa, P.: On the validity of layered models of ocean dynamics and thermodynamics with reduced vertical resolution. *Dyn. Atmos. Oceans* **29**:1–40, 1999.