# A High-Pressure High Strain Rate Elastic-Viscoplastic Model for Cementitious Materials

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Summary A comprehensive experimental study aimed at characterization of the combined effects of high confinement and high strain rate on the deformation and strength of cementitious materials was conducted. Quasi-static triaxial compression tests for confining pressures ranging from 0 to 500 MPa were performed. Dynamic tests for strain rates in the range 60/s to 160/s under both unconfined and confined conditions were conducted using a split Hopkinson pressure bar. A new elastic/viscoplastic model was developed.

The combined effects of high confinement and high strain rate on the mechanical response of two cementitious materials, mortar and concrete, have been investigated. Mortar is defined as simply a mixture of cement, fine-grained sand, water and binders, whereas concrete is defined as a mixture of the same materials plus coarse aggregate. For this study, a nominally 52.5 MPa unconfined compressive strength mortar, and 60 MPa unconfined compressive strength concrete with a 9.5 mm chert aggregate were selected.

The effects of confining pressure on cementitious materials tested at quasi-static strain rates have been extensively studied. Effects of strain rate on the compressive and tensile properties of cementitious materials have also been reported (e.g. Ross et al. [1] and, Malvar and Ross [2] respectively). A very limited set of data on strain rate effects on dynamic shear strength of cementitious materials is given by Schmidt and Ross [3].

Data on the combined effects of confining pressure and strain rate are rather limited due to the necessity of complicated test equipment and the complexity of test procedures. Some of the earliest dynamic confined tests were performed by Malvern et al. [4] using a 7.62 cm diameter Split Hopkinson Pressure Bar (SHPB) and a pressure cell for applying confining pressure in the range 0-6.9 MPa. Using the same device as Malvern et al. [4], confined dynamic tests were performed on strain gage instrumented mortar and concrete test specimens at strain rates ranging from 60/s to about 160/s under both unconfined and confined conditions. Quasi-static triaxial compression tests at a strain rate of $10^{-6}$/s were also conducted on the concrete material for confining pressures ranging from 0 to 500 MPa.

For mortar, dilatancy has been observed at high levels of the principal stress difference for both dynamic and quasi-static conditions. The unconfined dynamic compressive strengths are approximately double those of the quasi-static compressive strengths. Most of the confined SHPB mortar specimens showed very little damage post-test. For concrete, the unconfined dynamic strength is as high as 1.5 times the quasi-static strength, the material generally exhibiting far more cracking under similar loading conditions than was observed in mortar. The confined dynamic tests showed similar stress-strain response as the quasi-static tests conducted at the same level of confinement. For both materials, a decrease in strain rate sensitivity with increasing confining pressure was observed. Figure 1 illustrates this phenomenon for concrete. In the figure, results are given in terms of a Dynamic Increase Factor (DIF), or the ratio of the quasi-static to dynamic strength.

For the concrete, quasi-static hydrostatic tests conducted up to a pressure of 0.5 GPa allowed for the accurate determination of the dependence of the bulk modulus on pressure and the correct estimation of the material's compaction properties when subjected to pressures in the range encountered in dynamic events. For confined quasi-static conditions, the material exhibited hardening behavior up to failure. Both compressibility and dilatancy regimes of the volumetric behavior were observed, the dilatancy threshold being highly dependent on the level of confinement. A new elastic/viscoplastic model that captures compressibility and dilatancy, as well as strain rate effects has been developed for concrete. As a general framework, the elastic-viscoplastic formulation of Cristescu [5] was chosen due to its apparent capability to capture the behavior of interest and the fact that there are no a priori limitations or restrictions regarding the specific expressions of the yield function and viscoplastic potential. However, it was found that in order to capture the peculiarities of the volumetric response, namely the reduced dilatancy at very high pressure, the hypothesis of existence of a viscoplastic potential has to be abandoned. The formulation does not assume a decoupling between shear and volumetric effects, and does not require a separate equation of state in order to handle the pressure volume relationship. A new flow rule was proposed using general representation theorems for tensor functions (Cazacu et al. [6]). The general form of the constitutive equation is shown below, where $\varepsilon$ is strain, $\sigma$ stress, $G$ the shear modulus, $K$ the bulk modulus, $p$ the mean normal stress, $\delta_{ij}$ the Kronecker delta function, $k_T$ a viscosity parameter, $W$ irreversible stress work, $H$ the yield function, $t$ is time, and $N$ the strain rate orientation tensor function. In the equation, the over dot is used to indicate differentiation with respect to time, while the symbols, $\langle\ \rangle$, represent

$$\dot{\varepsilon}_{ij} = \frac{\dot{\sigma}_{ij}}{2G} + \left(\frac{1}{3K} - \frac{1}{2G}\right)\dot{p}\delta_{ij} + k_T \left\langle 1 - \frac{W(t)}{H(\sigma_{mn})} \right\rangle N(\sigma_{ij})$$

the so-called Macaulay bracket, used to denote the positive part of a function (i.e. $\langle A\rangle = \frac{1}{2}\left(A+|A|\right)$). Detailed procedures for determination of the constitutive functions were developed (Schmidt [7]). Figures 2 through 4 present comparisons between the model predictions and experimental data. The results show that the proposed model describes with very good accuracy the behavior of concrete over a broad range of pressures.

## References

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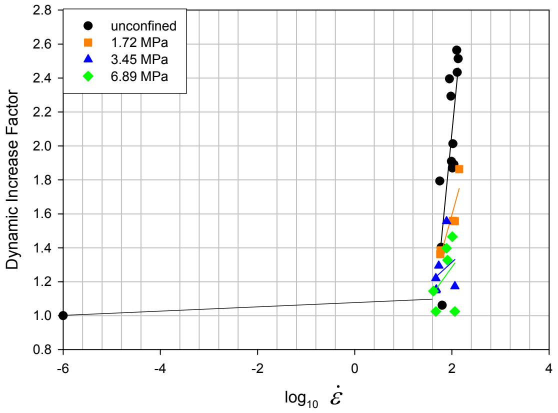

Figure 1: Effect of confinement on Dynamic Increase Factor (DIF) for concrete

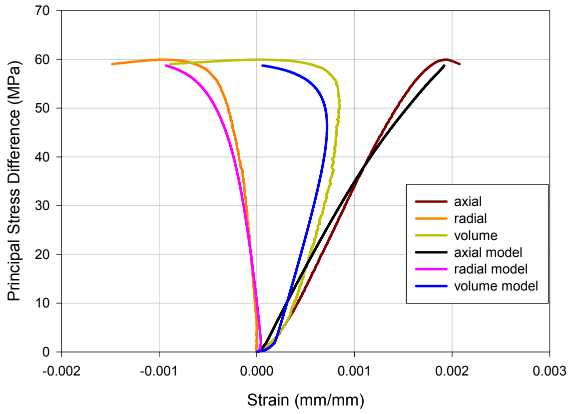

Figure 2: Experimental/Theoretical comparison of uniaxial compression test for concrete

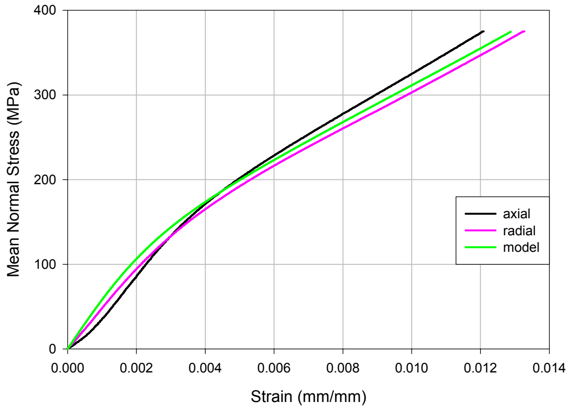

Figure 3: Experimental/Theoretical comparison of 375 MPa hydrostatic compression test

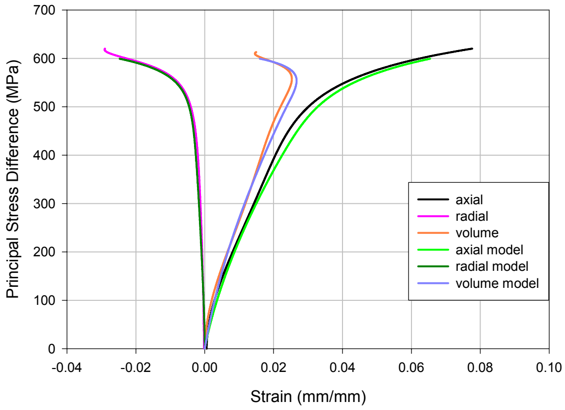

Figure 4: Experimental/Theoretical comparison of 375 MPa deviatoric compression test

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