UPPER AND LOWER BOUNDS OF ELECTRIC INDUCTION INTENSITY FACTORS FOR MULTIPLE PIEZOELECTRIC CRACKS BY THE BEM

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Summary The upper and lower bounds for the electric induction intensity factors for multiple piezoelectric cracks are obtained by the boundary element method using the impermeable and permeable crack solutions. The numerical Green's function for the crack is developed by the analytical integration of the continuous distribution of the generalized dislocation dipoles. The Green's function has the $1/\sqrt{r}$ generalized stress singularity and no post process for the intensity factor determination is needed.

INTRODUCTION

The coupling of mechanical and electrical behaviors of the piezoelectric materials has find its applications to sensors (e.g., sonars), actuators (e.g., ultrasonic cleaners, ultra-precision positioners, ink jet print heads), signal transmitters (e.g., cellular phone, remote car opener), and surface acoustic wave devices to mention a few. The modern life style cannot be sustained without piezoelectric materials. However, they are plagued with the brittleness of the widely used piezoceramic materials. The lack of understanding and modelling tools of the piezoelectric fracture is limiting the further progress in the piezoelectric material based technology. This paper addresses issues on the crack surface electric boundary conditions and suggests the upper and lower bound approach in the determination of the electric induction intensity factors using the boundary element method.

CRACK SURFACE ELECTRIC BOUNDARY CONDITIONS

While the mechanical boundary condition (BC) on the crack surface is always traction-free, the electric boundary condition comes in different degrees of shielding the electric induction defined by the electric permeability. For a crack along the x_1-axis, the permeable BC

$$D_2^+ = D_2^-; \quad \Phi^+ - \Phi^- = 0, \tag{1}$$

does not shield the electric induction at all, where D_2 and Φ are the electric induction and the electric potential, respectively, with $\pm$ indicating the upper and lower crack surfaces. Meanwhile, the impermeable BC

$$D_2^+ = D_2^- = 0, \tag{2}$$

shields the electric induction completely. The permeable BC is correct if the crack is closed, while the impermeable BC is correct if the permittivity ε_c of the crack medium is zero. Since no medium has zero permittivity (the vacuum has the least permittivity $\varepsilon_0 = 8.854 \times 10^{-12} C/(Vm)$) and we consider open cracks, the both boundary conditions are not correct. Hao and Shen [1] proposed the semi-permeable BC

$$D_2^+ = D_2^-; \quad D_2^+(u_2^+ - u_2^-) = -\varepsilon_c(\Phi^+ - \Phi^-), \tag{3}$$

which is the consistent BC adopted in this proposal. Note that the semi-permeable BC is reduced to the impermeable BC when $\varepsilon_c = 0$ and to the permeable BC when $u_2^+ - u_2^- = 0$ and that the impermeable and permeable BCs set the bounds for the semi-permeable BC. The majority of the earlier works adopted the impermeable BC due to the convenience to obtain the analytical solution. This led to contradiction between the experimental results and the fracture prediction by the (negative) energy release rate. Results predicted by the permeable BC are in much better agreement with the experimental results than those by the impermeable BC. If the crack opening displacement is extremely small, the permeable BC may provide a good approximation despite its inconsistency. The shift toward the consistent semi-permeable BC was made gradually but slowly.

UPPER AND LOWER BOUNDS FOR THE INTENSITY FACTOR

While the solutions of the permeable and the impermeable cracks are linear, that of the semi-permeable crack is non-linear. Since the distribution of the electric induction on the crack surface, which is needed to determine the crack opening displacement and the electric potential jump across the crack, is unknown we need an iteration process to determine all unknowns. This is inherently a nonlinear process even though each step consists of the linear solver. This poses a serious difficulty for the complex multiple crack configurations that appear in the real life applications. Meanwhile, the analytical solution for the single semi-permeable crack suggests the electric induction intensity factor is bounded by those by impermeable and the permeable cracks. So even though we may not get the exact semi-permeable solution, we can still get the upper and lower bound solutions using the impermeable and permeable cracks.

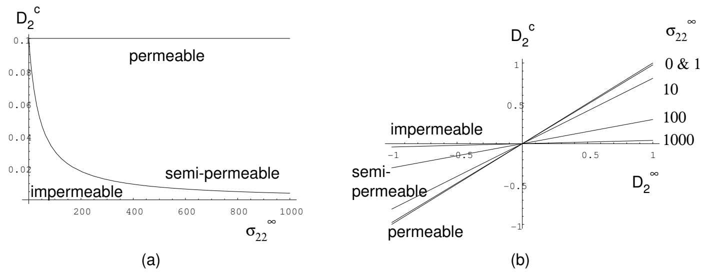

Figure 1. The crack face electric induction D_2^c for BSN with $H_{24} = 0$.

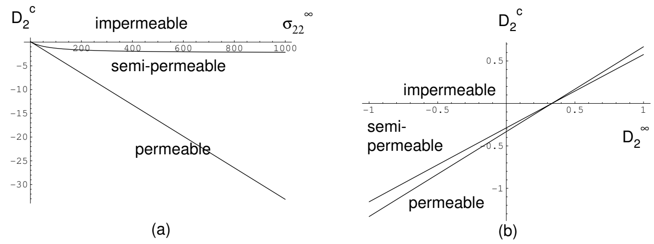

Figure 2. The crack face electric induction D_2^c for BSN with $H_{24} \neq 0$.

NUMERICAL GREEN'S FUNCTION FOR PIEZOELECTRIC CRACKS

While the FEM needs to model the entire domain surrounding a crack, the BEM only needs a single line to model the crack. It makes more sense to use the BEM for crack problems. As the demands for the analysis of more complicated crack configurations, such as multiple and curvilinear cracks, arise the advantage of the BEM over the FEM becomes clearly visible. We propose to use the BEM crack modelling strategy developed by Denda and Mattingly [2] for the general anisotropic solids. It uses the Green's function approach to multiple crack problems. The crack is modeled by the continuous distribution of the generalized dislocation dipoles with the $\sqrt{r}$ crack tip behavior and the resulting integration is evaluated analytically using the complex variable theory. The resulting crack element, called the whole crack singular element, offers a collection of mutually independent crack opening modes each of which has the $1/\sqrt{r}$ crack tip stress singularity. It serves as the numerical Green's function for the crack since the magnitudes of each mode must be determined numerically to satisfy the crack surface boundary condition. The solution strategies for the impermeable and permeable cracks are summarized as follows. (1) For the impermeable crack all the crack surface generalized traction components are set to zero. (2) For the permeable crack set the electric potential jump $\delta_4^{(m)}$ to zero and solve the problem by only applying the traction zero boundary condition on the crack. Since $\delta_4^{(m)} = 0$ we only need three traction free boundary condition on the crack surface to determine three unknown crack opening displacement components. Both (1) and (2) are linear procedures and can be solved by the BEM for multiple crack problems.

NUMERICAL RESULTS AND CONCLUSIONS

Figure 1 shows Variation of D_2^c $(\times 10^{-2} C/m^2)$ (a) as the function of σ_{22}^{∞} $(\times 10^7 N/m^2)$ for fixed $D_2^{\infty} = 0.1 \times 10^{-2} C/m^2$ and (b) as the function of D_2^{∞} for fixed $\sigma_{22}^{\infty} = (0, 1, 10, 100, 100) \times 10^7 N/m^2$; material is the Barium Sodium Niobate (BSN) with $H_{24} = 0$. Figure 2 shows the corresponding variation for the BSN with $H_{24} \neq 0$. (a) For fixed $D_2^{\infty} = 0.1 \times 10^{-2} C/m^2$ and (b) For fixed $\sigma_{22}^{\infty} = 10 \times 10^7 N/m^2$. Note that H_{24} is the measure of coupling between the crack opening δu_2 and the electric potential jump $\delta\phi$ and depends on the material constants and directions. These figures indicate that the impermeable and the permeable cracks set the bounds for the semi-permeable cracks.

References

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[2] Denda M., Mattingly, E.: The Whole Crack Singular Element for 2-D Boundary Element Analysis of Multiple Straight Cracks in the General Anisotropic Solids *Electronic J. Boundary Elements* (web site http://tabula.rutgers.edu/EJBE/) **1**:404-417, 2003.