NUMERICAL ANALYSIS OF STRAIN HARDENING AND PRESSURE SENSITIVITY EFFECTS ON *J*-INTEGRAL

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Summary *J*-integral is investigated for compact tension specimen for a strain-hardening pressure-sensitive material. Alower-bound plastic limit-load analysis is used with linear hardening assumed for the material. Pressure sensitivity, is accounted for by using the Drucker-Prager yield criterion. FE analysis was conduted using ABAQUS, with strain hardening modelled for typical hardening materials. Numerical and theoretical results are compared; *J* increases with increasing strain hardening and decreasing pressure sensitivity.

INTRODUCTION

While linear elastic fracture mechanics (LEFM) theory is utilized for estimation of fracture behaviour of materials in the elastic range, it is not adequate for large scale inelastic behaviour is encountered. To characterize the elastic-plastic fracture behaviour, Rice [1, 2] introduced the path-independent *J*-Integral around the tip of fracture notch. Considering a rigid perfectly-plastic material, Merkle and Corten [3] presented *J* estimation for compact tension specimens using a limit load analysis by considering the effects of the combined loading of axial force and bending moment applied to the remaining ligament of the specimens. Recently, Al-Abduljabbar and Pan [4] considered the effect of pressure sensitivity on the evaluated values of η and *J* for compact tension specimens with the same assumption of rigid perfectly-plastic material behaviour. Using finite element analysis, Kirk and Dodds [5] considered different equations used for estimating the η-factor, and highlighted the η dependency on strain hardening, which is not taken into account in the relevant standard (ASTM E 1290). In this work, we derive analytical expressions for factors of the *J*-Integral for a material with strain hardening by assuming a simple linear hardening model for the material. Moreover FE analysis is used to model the specimen with pressure sensitivity is accounted for by using the Drucker-Prager yield criterion for solid materials. The strain hardening behaviour is modelled for typical hardening materials as well as that of the simplified model using linear hardening curve which was used in the theoretical work. The effect of different hardening behaviours is also considered.

PLASTIC LIMIT ANALYSIS

Effects of the axial force and the bending moment acting on the upper half of the compact tension specimen are analysed by the use of a lower-bound approach whereby a single parameter can be identified to express the effect of the axial force. First, the material hardening behaviour is expressed according to Ludwik's expression,

$$\sigma = \sigma_0 + H\varepsilon^n , \tag{1}$$

where *H* and *n* are the hardening constants. Consider a compact tension specimen of perfectly plastic material as shown in Figure 1, loaded up to fully-plastic limit load P_0. Stress distribution in remaining ligament of the specimen is assumed as shown. The dimensionless parameter, α, is used as an indicator of the deviation of stress reversal point from the centre of the remaining ligament *b*. Another parameter, β, is used to account for the hardening effect. The slope of the linear hardening curve on the stress diagram is $\beta\, \sigma_0 / c$, and stresses σ_A and σ_B are determined as

$$\sigma_A = [1+\beta(1-\alpha)]\sigma_0 ; \tag{2}$$

$$\sigma_B = [1+\beta(1+\alpha)]\sigma_0 . \tag{3}$$

The force and moment balance around stress reversal point are enforced, from which an expression for α is obtained:

$$\alpha = \left[(1+\beta)^2 \left(\frac{a}{c} + 1 \right)^2 + \left(1 + \frac{2}{3}\beta \right) \right]^{1/2} - (1+\beta)\left(\frac{a}{c} + 1 \right). \tag{4}$$

An expression for the *J*-Integral is determined by namely two parts: the real work and complimentary work done on the specimen due to the applied load, as follows,

$$J = \int_0^{\Delta} \frac{1}{2P_0} \frac{\partial P_0}{\partial c} P d\Delta + \int_0^{\Delta} \frac{1}{2} \frac{\partial \theta}{\partial c} \left[\frac{\partial \theta}{\partial \Delta} \right]^{-1} dP ; \tag{5}$$

where θ is the angle of rotation defined from the relation between applied displacement Δ and crack-tip opening displacement δ. The coefficients of the first and second terms in Equation (5) are assigned to η and $\eta *$ respectively. From the relations of α to geometry and *J* definition, the parameter η is derived as

$$\eta = 2 \frac{\phi + (1+\beta)\phi}{\phi + \alpha^2} , \tag{6}$$

where ϕ=1+2 β/3. For perfectly-plastic materials, β=0; so Eq. (7) reduces to perfectly-plastic material solution.

Discussion

The finite element code ABAQUS is used to model the specimen of a pressure-sensitive strain-hardening material. Pressure sensitivity is modeled by using the Drucker-Prager yield criterion wherein the generalized equivalent stress is represented as a combination of the equivalent stress and the hydrostatic stress by the means of a pressure sensitivity factor. Strain hardening behavior is modeled for typical hardening materials as well as that of the simplified model using linear hardening curve which was used in the theoretical work. Plain-strain reduced-integration elements CPER8 are used for the model. The specimen dimensions are according to standard test specimen dimensions from ASTM E 1290. Consider the change of η with respect to the strain-hardening coefficient β for different cases of normalized crack length as depicted in Figure 2. It shows that for specimens with shallow cracks, an increase in the hardening factor β results in considerable change in η.

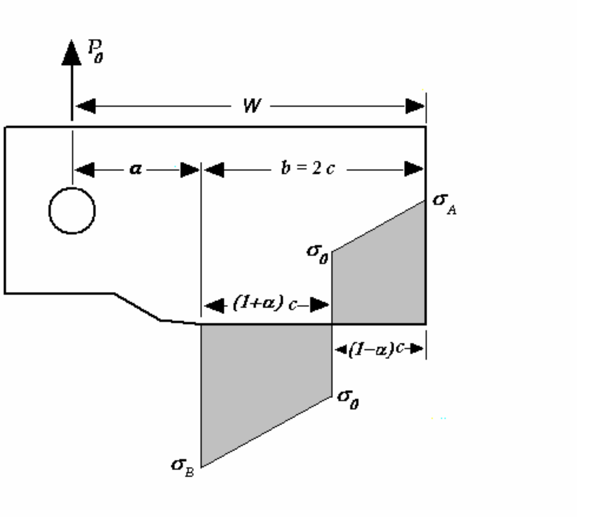

Figure1: CT specimen and stress distribution in remaining portion.

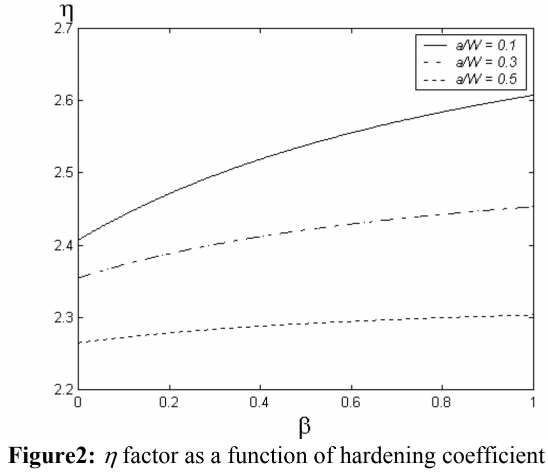

Figure2: η factor as a function of hardening coefficient β.

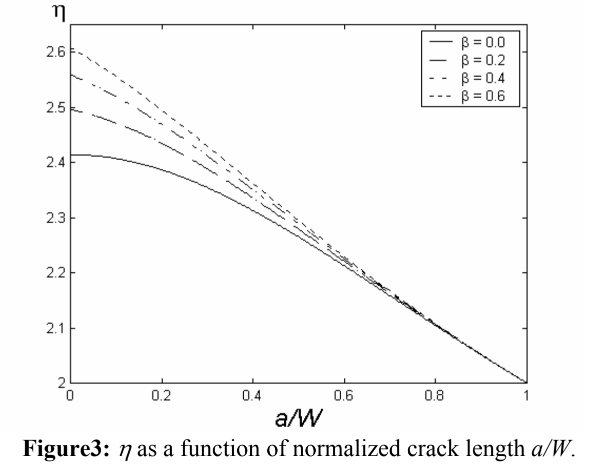

Figure3: η as a function of normalized crack length *a/W*.

Figure 3 shows the η factor as a function of the normalized crack length *a/W*. We can see from the plot that the effect of hardening decreases as the crack gets deeper and η converges to the value of 2 because for deeply cracked specimens, η approaches 2 regardless of the material constitutive behavior [6].

CONCLUSION

Results obtained from numerical and theoretical work show that as the material strain hardening increases, the *J*-Integral increases especially at low values of crack length. The effect of pressure sensitivity, on the other hand, tends to reduce the value of *J*. These results are relevant to ductile hardening solids with pressure-sensitive yielding such as nodular and malleable cast irons and even some steels. The forgoing analysis permits an improved estimation of the fracture parameter η for a material with strain hardening over the results reported for rigid perfectly plastic materials. Since most metals exhibit strain-hardening behavior, it is most suited to investigate the change in the fracture parameters due to the material hardening under further loading beyond the yield limit.

References

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