AXIAL DECAY OF TIME HARMONIC END PERTURBATIONS IN PRESTRETCHED HYPERELASTIC PLATES

Baruch Karp* and David Durban**
*Dept. of Mechanical Eng., Ben-Gurion Univ. of the Negev, P.O.B. 653, Beer Sheva 84105, Israel
**Faculty of Aerospace Engineering, Technion-Israel Institute of Technology, Haifa 32000, Israel

Extended Summary

Existing studies on axial decay rates, of end loads applied to long plates, concentrate almost exclusively on elastostatic response [1], with just a limited number of publications devoted to elastodynamic response. An early paper by Boley [2] showed, for a simple linear elastic model, that the extent of a Saint-Venant zone increases with the rate of applications of end loads. More recent papers have examined axial decay of transient response induced by various non-harmonic [3] and harmonic [4] time histories of end loads, again within the framework of linear elastodynamics.

Here, we present a detailed study of axial decay of time harmonic end perturbations applied to a prestrained plate. The setting is that of a hyperelastic semi-infinite plate, uniformly stretched, under plane strain conditions, in the axial direction. A time harmonic end perturbation is then applied at the edge and we examine axial decay rates of the resulting eigenmodes.

The mathematical formulation is within the framework of continuum elasticity, accounting for large deformations, with various boundary conditions imposed over the long faces of the plate. A separation of variables solution is assumed with due distinction between symmetric and antisymmetric eigenmodes. Our main interest is in axially decaying modes and in propagating (traveling modes).

Compliance with boundary data generates transcendental equations for the wave-numbers at a given frequency, initial stretch, material properties and boundary conditions. We have examined four sets of boundary constraints which include free, clamped, sliding and inextensional boundaries. Material behavior is modeled by hyperelastic strain energy functions, for three different materials (with experimentally determined parameters), over a wide range of initial stretch. Frequency maps are displayed in some detail to assess the various sensitivities of the wave number. We have studied the influence of initial stretch λ, exciting frequency Ω, instantaneous moduli and boundary conditions, with the main findings supported by asymptotic expansions. A rich and complex space of results has been exposed, yet it appears that some observations of general nature can be offered.

Apart from a finite limited number of propagating eigenmodes, all eigenmodes decay with distance from the perturbed end. The number of propagating eigenmodes increases with frequency Ω, yet at low frequencies there are just a few of them. No attempt is made in this study to address the issue of completeness (and orthogonality) of the eigenmodes. Assuming however that any end disturbance can be spectrally decomposed into a linear combination of all eigenmodes, we find that only a limited number of components will not decay in the axial direction. These propagating waves violate of course the principle of Saint Venant. On the other hand, all other eigenmodes, which are infinite in their number, exhibit a clear axial attenuation. That pattern suggests perhaps a restricted validity of Saint-Venant's principle in elastodynamics. In the vicinity of unstable states (necking and buckling) dynamic response of the plate exhibits a strong sensitivity to initial strain. The results of this study are applicable to design of composite laminated structures and to dynamic identification of internal defects.

References

[1] Horgan C.O., Knowles J.K.: Recent Developments Concerning Saint-Venant's Principle. In: Advances in Applied Mechanics, 23, 176-269, 1983.

[2] Boley B.A.: The Application of Saint-Venant's Principle in Dynamical Problems. *J. Appl. Mech* **22**, 204-206, 1955.

[3] Novozhilov V.V., Slepian L.I.: On Saint-Venant's Principle in the Dynamics of Beams. *PMM* **29**, 261-281, 1965.

Grandin H.T., Little R.W.: Dynamic Saint-Venant Region in a Semi Infinite Elastic Strip. *J. Elasticity* **4**, 131-146, 1974.