DYNAMIC ANALYSIS OF GRADIENT ELASTIC SOLIDS BY BEM

Demosthenes Polyzos*, Katerina G. Tsepoura*, Dimitri E. Beskos**
*Department of Mechanical and Aeronautical Engineering, University of Patras, GR-26500 Patras, Greece
**Department of Civil Engineering, University of Patras, GR-26500 Patras, Greece

Summary A frequency domain boundary element method is presented for the harmonic and transient dynamic response analysis of three-dimensional solids exhibiting linear elastic material behavior coupled with microstructural effects taken into account with the aid of a simple gradient elastic theory. Numerical examples are presented and the gradient elasticity effect on the response is assessed.

Extended Summary

The classical theory of linear elasticity cannot describe satisfactorily the mechanical behavior of linear elastic materials with microstructure, such as polymers, polycrystals or granular materials. In these materials microstructural effects are important and the state of stress has to be defined in a nonlocal manner. One way of successfully modeling these microstructural effects in a macroscopic manner is by using higher-order strain gradient theories.

In this work a simple form of the strain gradient theory due to Mindlin [1, 2] is employed of the form

$$\tilde{\boldsymbol{\sigma}} = \tilde{\boldsymbol{\tau}} + \tilde{\mathbf{s}} \tag{1}$$

$$\tilde{\boldsymbol{\tau}} = 2\mu\tilde{\mathbf{e}} + \lambda(\nabla\cdot\mathbf{u})\tilde{\mathbf{I}}, \qquad \tilde{\mathbf{e}} = (\nabla\mathbf{u} + \mathbf{u}\nabla)/2 \tag{2}$$

$$\tilde{\mathbf{s}} = -\nabla\cdot\tilde{\boldsymbol{\mu}}, \qquad \tilde{\boldsymbol{\mu}} = g^2\nabla\tilde{\boldsymbol{\tau}} \tag{3}$$

where $\tilde{\boldsymbol{\sigma}}$ is the total stress tensor, $\tilde{\boldsymbol{\tau}}$ and $\tilde{\mathbf{s}}$ are the Cauchy stress and relative stress tensors, respectively, $\tilde{\mathbf{e}}$ is the strain tensor, $\mathbf{u}$ is the displacement vector, $\tilde{\boldsymbol{\mu}}$ is the double stress tensor, g^2 is the volumetric strain gradient energy coefficient and λ and μ are the Lamè constants. The above constitutive model has been successfully used in a boundary element formulation for the static analysis of 2-D and 3-D gradient elastic solids (Polyzos et al [3]). In this work this methodology is extended to the dynamic case.

The frequency domain equation of motion of a gradient elastic body with material behavior described by eqs (1) – (3) takes the form

$$\mu\nabla^2\mathbf{u} + (\lambda+\mu)\nabla\nabla\cdot\mathbf{u} - g^2\nabla^2\left(\mu\nabla^2\mathbf{u} + (\lambda+\mu)\nabla\nabla\cdot\mathbf{u}\right) + \mathbf{f} + \rho\omega^2\mathbf{u} = 0 \tag{4}$$

where $\mathbf{u}$ now represents the displacement amplitude, ω is the excitation frequency, ρ the mass density and $\mathbf{f}$ the body force.

A variational statement is used to re-derive the equation of motion (4) as well as to determine all possible classical and non-classical (due to gradient terms) boundary conditions of the general boundary value problem in the frequency domain.

The gradient frequency domain elastodynanic fundamental solution is explicitly derived and used to construct the boundary integral representation of the problem with the aid of a reciprocal integral identity. In addition to a boundary integral representation for the displacement, a boundary integral representation for its normal derivative is also necessary for the complete formulation of a well posed problem. These two boundary integral equations for zero body forces and a smooth boundary S read

$$\frac{1}{2}\mathbf{u} + \int_S \left(\mathbf{P}^*\cdot\mathbf{u} - \mathbf{U}^*\cdot\mathbf{p}\right)\mathbf{d}S = \int_S \left[\mathbf{Q}^*\cdot\mathbf{R} - \mathbf{R}^*\cdot(\partial\mathbf{u}/\partial n)\right]\mathbf{d}S \tag{5}$$

$$\begin{aligned}\frac{1}{2}(\partial\mathbf{u}/\partial n) + \int_S \left[(\partial\mathbf{P}^*/\partial n)\cdot\mathbf{u} - (\partial\mathbf{U}^*/\partial n)\cdot\mathbf{p}\right]\mathbf{d}S = \\ = \int_S \left[(\partial\mathbf{Q}^*/\partial n)\cdot\mathbf{R} - (\partial\mathbf{R}^*/\partial n)\cdot(\partial\mathbf{u}/\partial n)\right]\mathbf{d}S\end{aligned} \tag{6}$$

where $\mathbf{u}$ and $\mathbf{p}$ are the displacement and traction vector amplitudes, n is the unit normal vector and $\mathbf{U}^*, \mathbf{P}^*, \mathbf{Q}^*$ and $\mathbf{R}^*$ are kernels of the gradient elastodynamic fundamental solution in the frequency domain.

The discretization of the body, restricted only to its boundary, is accomplished with the aid of surface quadratic quadrilateral boundary elements. Thus, eqs (5) and (6) are written in discrete form and after collocation and employment of the boundary conditions, the resulting system of equations is solved numerically for the given operational frequency to obtain the harmonic response. For the transient excitation case, the problem is solved in the frequency domain for a sequence of values of the frequency parameter and the transient response is obtained in the time domain by a numerical inversion of the frequency domain solution through the fast Fourier transform algorithm.

Two numerical examples serve to illustrate the method, demonstrate its accuracy and assess the gradient effect on the response. The two examples deal with the response of a spherical cavity embedded in an infinite gradient elastic medium to a uniform external pressure a) harmonically varying with time and b) of the step pulse type. The accuracy of the numerical results as compared to those of the analytic solution obtained by the authors is excellent. It is also observed that the response decreases with increasing values of the gradient coefficient in both cases.

References

[1] Mindlin R.D.: Microstructure in Linear Elasticity. *Arch. Rat. Mech. Anal.* **10**: 51-78, 1964.

[2] Mindlin R.D.: Second Gradient of Strain and Surface Tension in Linear Elasticity. *Int. J. Solids Struct.* **1**: 417-438, 1965.

[3] Polyzos D., Tsepoura K.G., Tsinopoulos S.V., Beskos D.E.: A Boundary Element Method for Solving 2-D and 3-D Static Gradient Elastic Problems, Pt I: integral formulation, Pt II: numerical implementation. *Comp. Meth. Appl. Mech. Engngn* **192**: 2845-2873 & 2875-2907, 2003.