

TRANSIENT DISPLACEMENT OF VISCOELASTIC LIQUIDS BY AIR

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Summary We examine the transient displacement by air of viscoelastic liquids, which occupy axisymmetric tubes either completely or partially, as it arises in the Gas-Assisted Injection Molding (GAIM) Process. For the simulations we combine our recently advanced quasi-elliptic grid generation scheme for computing the highly deforming liquid boundaries, the mixed finite element technique and the Discontinuous Galerkin method for computing the viscoelastic stresses. In our parametric study we examine the effects of the elastic and inertia forces, and the solvent to polymer viscosity ratio. Results using the PTT and Giesekus constitutive models show that the thickness of the remaining film increases as the Deborah number increases and remaining fluid fractions greater than 0.60 (the Newtonian limit) arise, in agreement with experiments. Increasing the viscosity ratio decreases the effects of elasticity. Similar is the effect of the Reynolds number which may also result in variable film thickness and a tip splitting instability.

INTRODUCTION

The displacement of a fluid by a gaseous phase is a problem of fundamental and practical importance. There are many engineering applications and physical operations such as the enhanced oil recovery, the pulmonary airway reopening, and the Gas-Assisted Injection Molding, which are based on the displacement of a liquid from a conduit. Apart from its more recent technological importance, the steady displacement of liquids in circular or non circular geometries has attracted the interest of many researchers for a long time and for various basic scientific reasons [1-3]. Quite recently, Dimakopoulos & Tsamopoulos [4-6] initiated a systematic effort in order to examine the transient gas-penetration phenomenon inside straight and suddenly constricted tubes, which are completely filled with a liquid. They examined both Newtonian and viscoplastic materials for a wide range of the dimensionless parameters.

The purpose of this study is to take into consideration the effect of viscoelasticity on the transient displacement process. To this end, we use simple but realistic constitutive equations such as the PTT [7] and Giesekus [8] models, which also allow us to also make simple physical interpretations.

PHYSICAL MODEL

The flow geometry is a cylindrical tube completely or partially filled with a liquid, while in the latter case the rest of the tube towards its right end is occupied by air. The liquid is incompressible and viscoelastic of total dynamic viscosity μ , density ρ and relaxation time λ and sticks to the tube wall, forming a three phase contact line at the left tube entrance. At start up, the air pressure on the left tube entrance is increased abruptly to \bar{P}_{ext} and displacing of the liquid is initiated. The radius and the length of the tube are denoted by a and b , respectively. Scaling lengths with a , pressure with the applied air pressure, \bar{P}_{ext} , velocities with $a^2 \bar{P}_{ext} / (\mu b)$, the following dimensionless groups arise: the Reynolds number, $Re_p = \rho a^3 \bar{P}_{ext} / (\mu b^2)$, the externally applied pressure, $P_{ext} = a^2 \bar{P}_{ext} / (b \sigma)$, where σ is the interfacial tension, the Deborah number, $De_p = a \lambda \bar{P}_{ext} / (b \mu)$, and the solvent to total viscosity ratio, $\beta = \mu_s / \mu (< 1)$. The transient motion of the liquid within the tube is governed by the momentum balance equations and the mass conservation law in their axisymmetric form. To determine the viscoelastic extra stress, we must introduce a constitutive law to the above set of equations. As such we use a differential model of the general dimensionless form:

$$De_p \overset{\diamond}{\underline{\underline{\tau}}}_p - 2(1 - \beta) \underline{\underline{D}} + \underline{\underline{g}}(\underline{\underline{\tau}}_p) = 0 \quad (1)$$

where $\underline{\underline{g}}(\underline{\underline{\tau}}_p)$ is a function of the polymeric stress, which goes to zero as $\underline{\underline{\tau}}_p$ approaches the same value and $\underline{\underline{D}}$ is the rate of strain tensor. Eq. (1) represents a number of well-known constitutive models including the Oldroyd-B, the Phan-Thien & Tanner (PTT) and the Giesekus models. The symbol \diamond over the elastic stress tensor denotes the Gordon-Schowalter time-derivative. These equations are subject to the usual boundary conditions on the velocity field. In particular, the velocity field is bounded at the centerline, and (a) in a completely occupied tube, open boundary conditions are applied at the outflow surface and at the tube wall the liquid is assumed to stick on it, while (b) in a partially occupied tube, a seconding moving boundary arises downstream and the liquid follows a Navier-type slip condition to accommodate the singularity at the moving contact line. Along any free surface, the velocity field satisfies a local balance between viscous stresses, surface tension and gaseous pressure and is determined by a kinematic condition.

NUMERICAL APPROACH

The governing equations, the momentum and mass balances, the constitutive equations and the two components of the

indispensable for this problem quasi-elliptic transformation ([6]) are converted to a set of Differential Algebraic Equations (DAEs) by invoking Galerkin's principle. The velocity vector, pressure and position vector are approximated by piecewise continuous basis functions, the velocity by biquadratic Lagrangian polynomials, while the coordinates of the mesh nodes and the pressure unknowns by bilinear functions of the local coordinates in an element. Due to its hyperbolic nature, the polymeric part of the stress is approximated using the Discontinuous Galerkin (DG) method ([9]). The set of DAEs are integrated using the backward Euler method, combined with a forward one for the prediction of solution at the new time instance and the adjustment of the time step. The resulting set of equations ($O(10^5)$) is solved by a Newton-Raphson/Non-linear Gauss-Seidel iteration scheme for separately solving the flow from the mesh generation equations until convergence to a specified tolerance of 10^{-9} for the absolute error norm of the residual vector

RESULTS AND DISCUSSION

The figure depicts the penetrating air bubble and contour lines of τ_{pzz} in the displaced Giesekus liquid which initially occupied completely the undulating tube. The snapshot corresponds to time $t'=2.114$, when the bubble tip has covered a distance equal to six radii and a thick film of liquid has been deposited on the tube wall. The latter seems to follow the shape of the wall with a small phase difference. We note the smoothness and clarity of the results, despite the discontinuous approximation of the stress tensor confirming the high accuracy of our computations. Due to the moderate value of elasticity and the finite value of β , τ_{pzz} takes non-zero values around the front of the bubble and downstream in the whole part of the tube. Examining more closely the stress field around the bubble front we distinguish two different regions, one compressive around the bubble tip, where τ_{pzz} attains its minimum value, -0.409, and another one extensional, where τ_{pzz} attains its locally maximum value, located at the side of the bubble front. Their absolute values decrease both radially and axially away from these two points (except for the expansive region of the tube walls where it attains its absolutely max value). The compressive region arises because the liquid resists the bubble-imposed deformation, while in the extensional region it tries to give again a flat front to the bubble. For penetration smaller than ~ 5 , velocities and stresses are spatially periodic in each cell due to the absence of inertial forces. We have obtained contour plots of the other stress velocity components for an extended range of parameter values. It is noteworthy that with our numerical scheme and as long as $\beta > 0$ we can compute for as high De_p as 5000, although in such cases we cannot simulate complete displacement of the liquid from the tube because of the appearance of steep boundary layers in stress around the bubble front. The thickness of the remaining film is increased up to 5% by increasing De_p over its Newtonian value, due to the normal stresses that arise in viscoelastic liquids, but not as much as in cases reported for non shear thinning (Boger) fluids and is decreased by increasing β or Re_p .

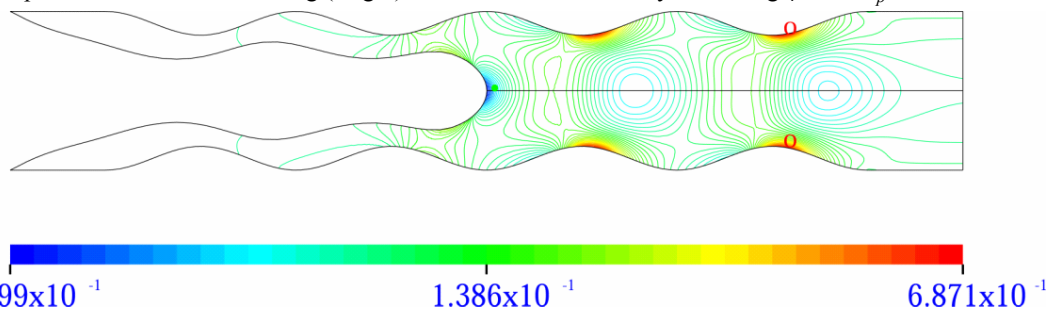


Figure. Contour lines of τ_{zz} in an undulated tube, which is completely filled with a Giesekus fluid at $t'=2.114$ for $(De_p, Re_p, P_{ext}, b/a, R_{min}/R_{max}, \beta, a_{GS}) = (0.5, 0, 8.33 \times 10^4, 12, 0.7, 0.5, 0.5)$.

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