

TRANSIENT SQUEEZE FLOW OF VISCOPLASTIC LIQUIDS

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Summary We examine the axisymmetric squeeze-flow, under creeping flow conditions, of viscoplastic materials placed between two parallel disks using the continuous constitutive model suggested by Papanastasiou and, more recently, the discontinuous Bingham model. This is the first transient simulation of such liquids and allows us to also determine the shape of the liquid/air interface, initially located inside the disks, as it is displaced towards their edge or beyond it. We employ the mixed finite element method coupled with a quasi-elliptic mesh generation scheme in order to follow these large deformations. The material yields in part of the domain as the disks approach each other, departing from the corresponding Newtonian solution. Unyielded material arises around the two stagnation points of flow at the disk centers verifying previous steady state calculations. The unyielded region increases with the Bingham number, but decreases with time. If wall-slip is present, the unyielded region decreases significantly and may even totally disappear.

INTRODUCTION

Viscoplastic materials behave as rigid solids for low stress values, whereas beyond some critical value of stress, called yield stress, they flow with viscosity that depends on the local shear rate. The deformation and flow of such materials is becoming increasingly important, since many multicomponent fluids, such as suspensions, emulsions, paints, foodstuffs, foams, slurries and pastes, which are frequently encountered in industrial processes, are viscoplastic. A typical experiment in order to characterize such materials is squeeze flow, in which a fluid sample is squeezed between two parallel disks. Although several researchers have studied flows of viscoplastic liquids in the past, they restricted it to steady state models [1]-[4]. This is the first truly transient simulation of such liquids and allows us to determine the shape of the liquid/air interface which was impossible with the quasi steady state models. The purpose of this study is to determine the liquid/air interface as well as the unyielded regions that arise as a function of relevant parameters of the problem. To this end, we use the continuous constitutive equation that has been proposed by Papanastasiou for viscoplastic liquids [5] as well as the discontinuous original Bingham model [6].

PHYSICAL MODEL

Liquid is placed in the gap between two parallel disks (see figure). The liquid is viscoplastic of zero shear viscosity η_0 and density ρ . Initially the distance between the two disks is $2L$, the liquid is at rest, under constant pressure and the free surface is flat. For $t > 0$ the disks approach each other with constant velocity V , displacing the liquid that exists between them. Scaling lengths with L , velocity with V the following dimensionless groups arise: the Reynolds number, $Re = \rho VL / \eta_0$ and the Bingham number, $Bn = \tau_y L / (\eta_0 V)$, where τ_y is the yield stress. The transient motion of the liquid within the disks is governed by the momentum-balance and the mass conservation law in their axisymmetric form. To determine the extra stress tensor we use the continuous model that has been proposed by Papanastasiou [5] for viscoplastic liquids.

$$\underline{\underline{\tau}} = \left[\eta_0 + \tau_y \frac{1 - e^{-n\dot{\gamma}}}{\dot{\gamma}} \right] \dot{\underline{\underline{\gamma}}}$$
, where $\underline{\underline{\tau}}$ is the viscous part of the total stress tensor, $\dot{\underline{\underline{\gamma}}}$ is the rate of strain tensor, $\dot{\gamma}$ its second

invariant and n a parameter. At large values of n the behavior of an ideal Bingham plastic is approached. As Burgos and Alexandrou [7] have mentioned extremely large values of n must be avoided. In order to determine the unyielded regions the criterion that was used is that fluid yields when the second invariant of the extra stress tensor $\|\underline{\underline{\tau}}\|$ is larger than the yield stress, $\|\underline{\underline{\tau}}\| \geq \tau_y$. These equations are subject to the usual boundary conditions on the velocity field. In particular, the velocity field is bounded at the axis and plane of symmetry. At the disk surface the fluid is assumed either to stick on it or wall slip is introduced via a parameter β (the no-slip condition results when $\beta \rightarrow \infty$). Along the free surface, the velocity field satisfies a balance between viscous stresses, surface tension and gaseous pressure.

NUMERICAL APPROACH

In order to solve these equations, we have chosen the mixed finite element method to discretize the velocity and pressure fields, together with a system of quasi-elliptic partial differential equations, capable of generating a boundary-fitted finite element discretization of the transient domain occupied by the liquid [8], [9]. The latter is an essential part of our method for capturing the large deformations of the free surface. We approximate the velocity vector with

biquadratic Lagrangian polynomials whereas the pressure and position vector with bilinear ones. The set of differential algebraic equations that arises is integrated using the backward Euler method, preceded by a forward Euler step in order to predict the new solution and then adjust the time step to optimize code performance. The resulting set of equations is solved by a Newton-Raphson/Non-linear Gauss-Seidel (Picard) iteration scheme for separately solving the flow from the mesh generation equations until convergence.

RESULTS

The figure depicts the domain between the upper disk and the plane of symmetry, from the axis of symmetry on the left side to the disk edge. In this example the liquid initially fills completely the space between the disks. Even from the very beginning of the motion of both disks towards the plane of symmetry the viscoplastic material yields everywhere, except for the area around the stagnation points of the flow. The figure also shows contour lines of the axial velocity component in the displaced viscoplastic liquid for $Bn = 100, Re = 0, \beta = 10^5, \varepsilon = 0.1$, where ε is the aspect ratio, $\varepsilon = L/R$. The snapshots correspond to dimensionless times $t=0, t=0.2, t=0.4$. We observe that the axial velocity u_z has its smallest (negative) values near the upper disk and monotonically increases towards zero at the plane of symmetry. At the axis of symmetry it remains equal to the velocity of the upper disk up to a point fairly close to the plane of symmetry because of the unyielded region that exists there. The size of this region decreases as time passes, because the correspondingly decreased distance between the disks generates a more intense shear field near $r=0$. In the process, material moves radially outward and emerges outside the domain between the disks while it undergoes apparent "extrudate swelling". Indeed, as the material exits the disks, it experiences smaller and smaller stresses becoming "frozen" and, thus, it retains the thickness it had at the instant it exited and before the disks moved further towards each other. The unyielded region decreases as ε or Bn decrease or the slip on the disk wall increases.

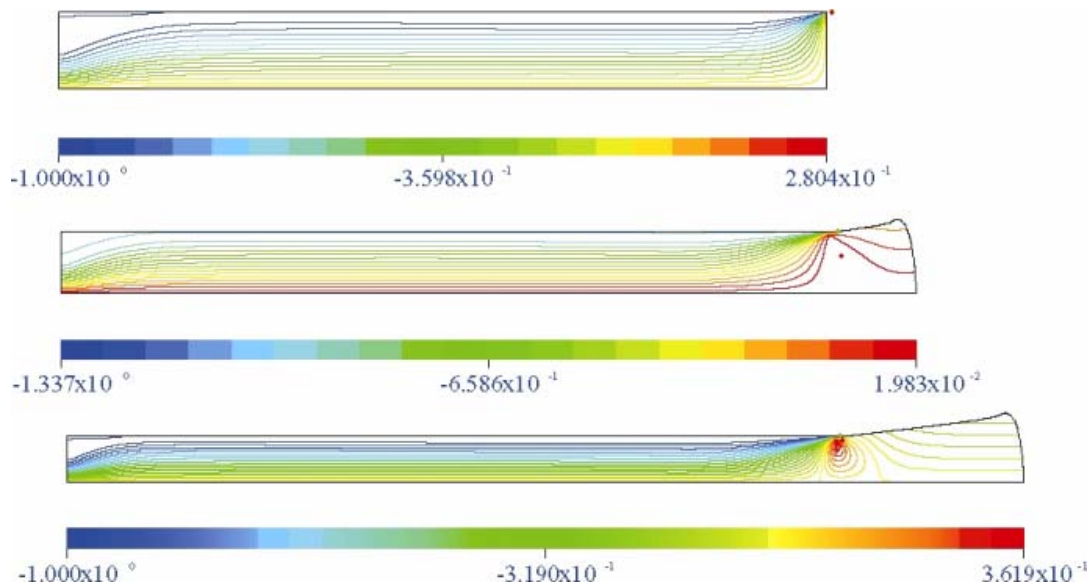


Figure Liquid/air interface shape and contour lines of axial velocity component at $t=0, t=0.2, t=0.4$ for $Bn = 100, Re = 0, \beta = 10^5, \varepsilon = 0.1$.

References

- [1] Smyrniotis D. N., Tsamopoulos J. A., "Squeeze flow of Bingham plastics", *J. Non Newtonian Fluid Mech.* **100** (2001) 165-190
- [2] O'Donovan, E.J., and Tanner, R.I., 'Numerical study of the Bingham squeeze film problem', *J. Non-Newt. Fluid Mech.*, **15**, 75-83 (1984)
- [3] Matsoukas A., Mitsoulis E., "Geometry effects in squeeze flow of Bingham plastics", *J. Non Newtonian Fluid Mech.* **109** (2003) 231-240
- [4] Jay, P., Magnin, A., and Piau, J.M., 'Viscoplastic fluid flow through a sudden axisymmetric expansion', *AIChE J.*, **47** (10), 2155-2166 (2001).
- [5] Papanastasiou T. C., "Flows of materials with yield", *J. Rheol.* **36** (1992) 389-407
- [6] Bingham, E.C., 'Fluidity and Plasticity', McGraw-Hill (1922)
- [7] Burgos G. R. & Alexandrou A. N., "On the determination of yield surfaces in Herschel-Bulkley fluids", *J. Rheol.* **43**(3) (1999) 463-483
- [8] Dimakopoulos Y. & Tsamopoulos J.A., "Transient displacement of a viscoplastic material by air in straight and suddenly constricted tubes", *J. Non Newtonian Fluid Mech.* **112** (2003) 43-75
- [9] Dimakopoulos Y., Tsamopoulos J.: A quasi-elliptic transformation for moving boundary problems with large anisotropic deformations, *J. Comp. Phys.*, **192**: 494-522, 2003c.