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VARIABLE-ORDER SINGULAR BOUNDARY ELEMENT FOR CALCULATION OF THREE-DIMENSIONAL STRESS INTENSITY FACTORS

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Summary A new variable-order singular boundary element has been developed for stress analysis of three-dimensional cracks and multi-material junctions. The singular element shape functions include the variable orders of singularity and the angular profiles of the field variables. The stress intensity factors are formulated as nodal unknowns and are obtained directly from the final system of equations without the need of post processing, such as three-dimensional J-integral.

INTRODUCTION

Stress singularities often occur in cracks and dissimilar material corners. Such singular stress fields are generally three-dimensional in character. The stress intensity factor is a function of the position along the line of singularity and not a constant as commonly determined in two-dimensional solutions. However, three dimensional stress singularity problems are more complicated to analyze compared to their two-dimensional counterparts. The finite element method [1, 2] and boundary element method [3] are commonly used. In this paper, we present a new variable-order singular boundary element that provides efficient and accurate solutions for three-dimensional stress singularity problems.

VARIABLE-ORDER SINGULAR ELEMENT FORMULATION

The formulation for the current three-dimensional singular boundary element is an extension of that of the two-dimensional singular element [4]. The singular field in the vicinity of the line of singularity is assumed to be dominated by a combination of two in-plane and one out-of-plane singular modes, with the stress intensity coefficients varying along the line of singularity. The variable orders and angular field profiles are obtained separately from asymptotic singularity analysis. These asymptotic solutions form the basis of the shape functions in the new singular element. Both asymptotic stress and displacement fields are represented simultaneously for a better accuracy even in a very coarse mesh.

The intrinsic coordinates (ξ, η) of the singular element are chosen such that the singular profiles lie in planes parallel to the η direction, while the stress intensities vary along the ξ direction. The tractions and displacements in the singular element are expressed as a combination of the singular fields and the non-singular polynomials as

$$t_i = \sum_{h=1}^{3}(a_{1h} + a_{2h}\xi + a_{3h}\xi^2)r^{\lambda_h - 1}p_{ih}(\theta) + a_4 + a_5\xi + a_6\eta + a_7\xi\eta + a_8\xi^2 + a_9\xi^2\eta$$

$$u_i = \sum_{h=1}^{3}(b_{1h} + b_{2h}\xi + b_{3h}\xi^2)r^{\lambda_h}q_{ih}(\theta) + b_4 + b_5\xi + b_6\eta + b_7\xi\eta + b_8\xi^2 + b_9\xi^2\eta$$

where t_i and u_i are the i^{th} component of the traction and displacement at the coordinate (ξ, η) respectively, and a_{jh}, a_j, b_{jh}, b_j (j=1 to 9) are independent coefficients to be determined. The first part in the equations represents the contribution from the singular fields in which the stress intensity coefficients are assumed to have a quadratic variation along ξ, and the order of singularity λ_h and the angular profiles $p_{ih}(\theta)$ and $q_{ih}(\theta)$ are assumed to be unchanged along the crack front. The second part includes non-singular polynomial terms which are obtained according to the sequence in the Pascal triangle. The coefficients are subsequently expressed in terms of the displacements, tractions and stress intensity factors at the nodes. The resulting expressions associated with these nodal quantities then give the shape functions for displacement and traction at each node.

The new singular surface elements replace the conventional elements adjacent to the line of singularity. Transition elements are not required because the singular elements are made naturally compatible with adjacent normal quadratic boundary elements. The stress intensity factors are formulated as nodal unknowns along the line of singularity and they can be directly obtained from the solution process of the boundary element method. The post-processing step commonly used in the literature, is not necessary in this case.

NUMERICAL EXAMPLES

First, a penny-shaped crack (radius c=0.08b) in a homogenous cube (Figure 1a) under remote tension is analyzed. Only one-eighth of the cube is modeled and a relatively coarse boundary element mesh is used. In the vicinity of the crack front, only six elements are used on each side of the crack front. The singular element gives accurate results (Figure 1b) for the mode I stress intensity factor K_I which is within 5% of the analytical value K_0 [5].

Next, the versatility of the singular element in handling various orders of singularity is illustrated using a bi-material system with a through-thickness inclusion under remote tension (Figure 2a). Singular fields occur at the fully bonded internal corner and the various orders of singularities are present: λ_1=0.798, λ_2=0.786 and λ_3=0.732. The equivalent stress intensity factors F are defined here along the horizontal interface: $t_y = r^{\lambda_1 -1}F_I\sigma_0 + r^{\lambda_2 -1}F_{II}\sigma_0$ and $t_z = r^{\lambda_3 -1}F_{III}\sigma_0$. The variations of the stress intensity factors through the thickness are determined. Figure 2b shows the results for the three modes. Plane strain conditions are also simulated by imposing symmetric boundary conditions on both the surfaces *z/t*=0 and *z/t*=0.5 and the results are included for comparison. It is found that the three-dimensional results for modes I and II are very close to the plane strain solutions for most part of the internal edge, while the intensity for mode III increases from the mid-plane *z/t*=0 to the free surface *z/t*=0.5.

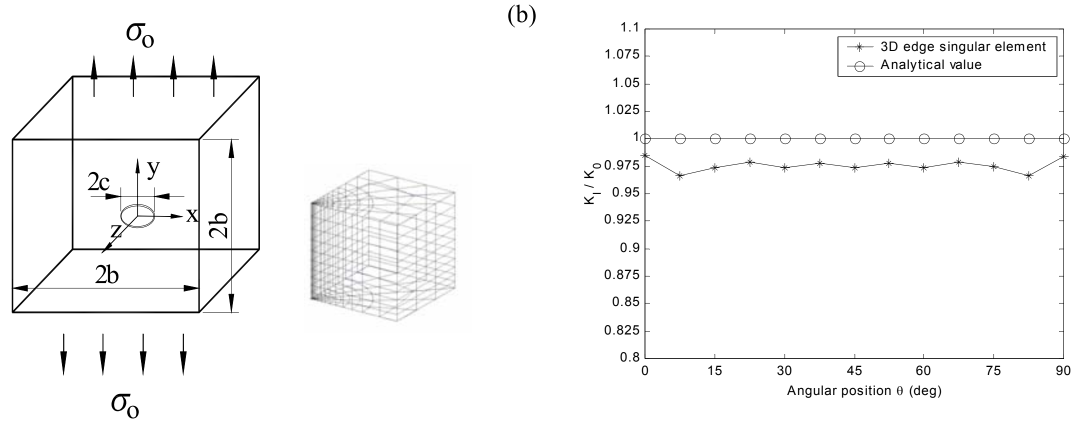

Figure 1: (a) Penny-shaped crack in homogeneous medium and the one-eighth model mesh. (b) Magnitude of stress intensity factor along crack front.

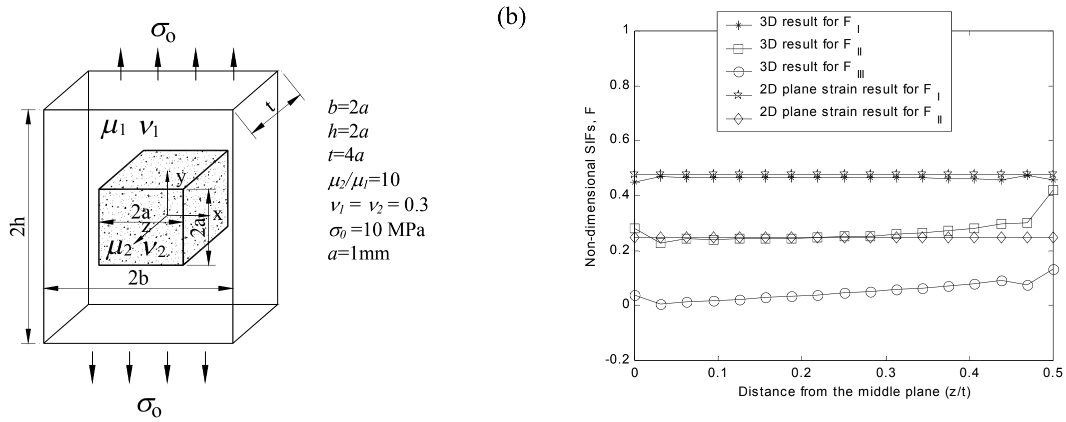

Figure 2: (a) Through-thickness bi-material inclusion. (b) Stress intensity coefficients along bi-material edge.

CONCLUSION

By incorporating both the variable orders and the angular variations, the new singular boundary elements can provide accurate solutions to three-dimensional stress singularity problems. The stress intensity factors which are normally difficult to determine using conventional numerical techniques are hereby easily computed with good accuracy using relatively coarse elements.

References

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