MODELLING OF SHORT FATIGUE CRACK GROWTH IN A METAL IN HCF RANGE

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S*ummary:* Modelling of short fatigue crack growth in a polycrystalline metal in high cycle fatigue range (HCF) is the subject of the paper. For this goal a fracture mechanics approach was adopted to predict stage-I and stage I-like crack growth. On the contrary, stage II crack growth is modelled using a probabilistic approach. Special attention is paid on the short crack behaviour under reversed cyclic torsion.

Description of the model

Short fatigue crack growth is analysed in three stages: as a small crack of a size comparable to the grain size (stage I), a microstructurally short crack (stage I-like) and a physically short crack (stage II). Stages I and I-like crack growth are strongly microstructurally depended and the crack advances under mode II. In the model the stage I crack growth is associated with one slip band and its growth is affected by a local elastic-plastic field resulted from a dislocation pile-up at the grain boundary. Assuming that a slip band of length L_{sb} and width h is approximated by a flattened elliptic crack of same length, the radius ρ of the crack tip and crack opening displacement (COD) δ_{II} $(h = \delta_{II} = 2\rho)$ then the amplitude of a local effective shear stress $\tau_{a,eff}\,(r,\theta)$ ahead of crack-slip band gets the form in polar coordinates:

$$\tau_{a,eff}(r,\vartheta) = \frac{2\cdot(m\cdot\tau_{a,appl} - \tau_o)\sqrt{\pi L_{sb}}}{\sqrt{2\pi(c\delta_{II} + r)}}\cos(\theta/2)\left[1 - 3\sin^2(\theta/2)\right] \qquad (1)$$

where: $\tau_{a,appl}$ is the amplitude of applied cyclic shear stress, τ_o is an intrinsic friction stress, m is the Taylor orientation factor, ρ is the radius of crack tip $\rho = c\delta_{II}$, r is a distance from the tip of crack-slip to a current point, c is constant geometric coefficient. The quantity θ is the angle between the slip band plane and the arrow to a current point.

The mechanism of stage I-like crack growth relies on repeated transfer of slip bands from grain to grain and crack nucleation on the slip band in the successive grains. Mikrocrack is blocked at the grain boundary until the shear stress at the crack tip achieves a relevant value allowing it to overcome the structural barrier and to spread slip band in the next grain on a slightly misoriented plane. Applying the Irwin's idea of modified crack length in the elastic-plastic fracture mechanics approach the local effective stress $\tau_{a,eff}\,(r)$ that acts at the tip of stage I-like crack in the plane of the crack is estimated as follows:

$$\tau_{a,eff} = \frac{K_{II\,eff}}{\sqrt{2\pi(c_e\delta_{II} + d^*)}} = \frac{1}{\sqrt{2\pi(c_e\delta_{II} + d^*)}}(m\cdot\tau_{a,appl} - \tau_{res})\frac{\sqrt{\pi a}}{\sqrt{1 - \frac{1}{2}\left(\frac{\tau_{a,appl}}{\tau_y}\right)^2}} \qquad (2)$$

where: a denotes the length of a physical crack, r_{pl} is the radius of reversed plastic zone, τ_y is the yield stress, $d^* = r_{pl}$, c is a constant coefficient, τ_{res} is the resistance of material to dislocation motion [1,2,3]. The condition required for crack growth from grain to a grain is that $\tau_{a,eff} = \frac{K_{IIC}}{\sqrt{2\pi d}}$, where K_{IIC} means a critical stress intensity factor for mode-II cracking, d is an average size of the grain in a polycrystalline metal. Interaction with a successive grain boundary leads to decreasing of crack growth rate. This trend in the growth rate is simulated by crack propagation equation:

$$\frac{da}{dN} = B(\tau_{a,eff})^n(L - a) \qquad (3)$$

where: B and m are material constants, L means a characteristic dimension in the matrix structure like as length of ferrite band or transient length between stage I and stage II crack growth.

Crack growth in the stage II (physically short crack) exhibits smaller dependence or even independence on the microctructural barriers. Crack growth rate shows the oscillations with a decreasing amplitude as the crack becomes longer up to a transient length. In the paper stage II crack growth is modelled using probabilistic approach. Fatigue process in an elastic-plastic material is regarded as a set of random values of crack length a_i at the related time t_i. Main equation of a dynamics of crack growth is the Fokker-Planck partial differential equation (4):

$$\frac{\partial U(a,t)}{\partial t} = -\lambda(t)\cdot \Delta a \frac{\partial U(a,t)}{\partial a} + \frac{1}{2}\lambda(t)\cdot \Delta a^2 \frac{\partial^2 U(a,t)}{\partial a^2} \qquad (4)$$

where *U(a,t)* is a probability density function of that event that a crack of the length *a* was found in a time *t*, Δa means crack length increment, λ(t) – frequency of cyclic load.

Constitutive equation is formulated in order to predict fatigue crack growth :

$$\frac{da}{dt} = C_{II}(CTOD)^n \qquad (5)$$

where CTOD means crack tip opening displacement defined on the base of the Dugdale-Barenblatt model, C_{II} and *n* are material constants.

Equation (5) gives the rule for estimating the crack length increment Δa. After substitution it to the equation (4) and making the assumption of the Gaussian shape of the probability density function *U(a,t)* one can derive the form for an average crack length and average crack growth rate at the time *t*.

Ability of the model proposed to predict the short crack growth has been verified using experimental data gathered for an hour glass shaped specimens of normalised 0.45% carbon steel fatigued under reversed torsion. Experimental curves that reflect the crack growth behaviour in the examined specimens indicated that the stage I and stage I-like crack growth cover the crack length range up to 130 μm. It relates to 20% of total life in each case of the tested specimens independently on the applied stress. Stage II crack growth comprises the crack length up to the threshold length =400 μm. It relates to 60-70% of total life in 0.45% carbon steel depending on the applied stress. Experimental and predicted plots of crack growth rate versus crack length and stress ratio N_i/N_f presents Fig. 1.

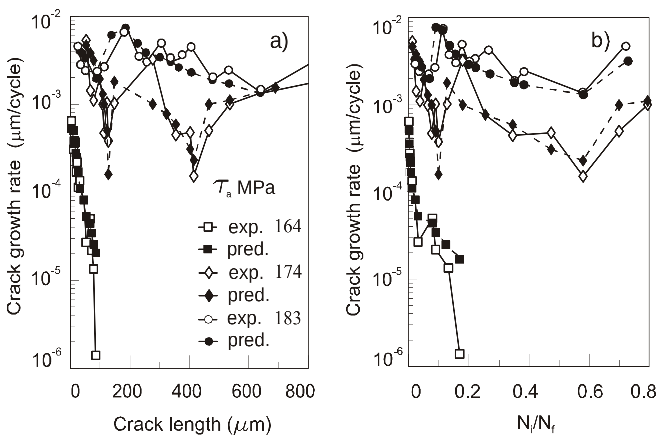

Fig. 1. Experimental and predicted plots of crack growth rate versus crack length and stress ratio Ni/Nf

CONCLUSION

The present model of short crack growth description carries significant modifications in the pre-existed models developed by authors [4, 5]. Taking into considerations the mechanism of crack growth in the stage I and stage II of short crack growth behaviour one can receive better prediction of crack growth as well as better estimation of fatigue life of a component. Experimental plots performed in the work indicate a long lasted period of short crack behaviour under reversed torsion in relation to the total life N_f.

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