CHARACTERISTICS OF VIBROACOUSTIC SIGNALS IN DIAGNOSING EARLY STAGES OF DEFECTS

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Summary We are considering certain properties of transformation of time series that register the actual physical process. Our goal is to define these basic features of the data analysis itself that ensure that the ”processing” applies to the properties of a phenomenon which generates the signal independently of its numerical representation and could be used for early recognition of arising new type factors and components of the source. It turns out that the information concerning that physical process we may find in some distribution of vector values determined by the time series. This invariant in described here sense characteristics of a vibroacoustic signal, could be used in diagnosing early stages of defects.

INVARIANTS OF NUMERICAL REPRESENTATION OF SIGNAL

When diagnosing the early stages of defects, particularly the initiation phases of fatigue-related cracks, the basic problem is the selection of the diagnostic parameters and methods of detection of diagnostic information. The required digitization of the signal may result in the threat of disregarding or minimizing the diagnostically useful signals. This may result in incorrect identification and location of a defect, and subsequently in formulation of incorrect forecast. Bearing in mind the importance of the problem, in this paper we present the results of analysis related to the possibility of determining the invariants of transformation of certain vibroacoustic signals. We are considering certain properties of transformation of time series that register the actual physical process. Our goal is to define these basic features of the data analysis itself that ensure that the ”processing” applies to the properties of a phenomenon which generates the signal independently of its numerical representation. The result of numerical processing of experimental data is intended to be the ”deciphering” of the information on quantitative relationships between the physical values of the examined mechanical model, which were constructed earlier in a model. In this paper we deal with the answer to the question concerning the type of information that we can obtain as a result of such an analysis and how to look for such information. We assume that the data available for numerical analysis, collected as a result of an experiment, are recorded in the form of a finite, ordered system of integers $\{x_k\}$. Since a finite sequence of such sets will still render an object of the same kind, thus we can infer that the potential sum (repetition) of experiments will not change the quality of the data available for processing. All the available information (in terms of ability to confirm the information through an experiment) on a physical phenomenon is hidden in $\{x_k\}$ objects, where $x_k \in Z$, $k \in \mathbf{N}$. Numerical processing is a transformation having the form of $x_k \xrightarrow{f} f(x_k)$, whose image, which emerges as a result of registration on a computer disk, is also a finite system of rational numbers. Function f, defined for sequences $\{x_{k_n}\}$, should reflect the feature of the signal while at the same time not being dependent on the randomly selected features of a sample. The fulfillment of this property will be called the invariance of f vis-a-vis random choices made during the experiment.
Due to natural reasons we distinguish three types of invariance depending on: sample length $\{x_{k=1}^N\}' = \{x_{k=1}^{N'}\}$, initial moment selection $\{x_k\}' = \{x_{k+m}\}$ and sampling step $\{x_k\}' = \{x_{kn}\}$. The function that fulfills the equation of

$$\lim_{N'\to\infty} f(\{x_k\}') = \lim_{N\to\infty} f(\{x_k\})$$

simultaneously for three such relations will be termed by us, in the same way as in the classical invariance theory, either the invariant function, while taking into account the influence of the affine group, or the globally affine invariant function. The functions that are invariant with regard to small changes of N, n, m parameters will be called by us the locally affine invariant functions.

CHARACTERISTIC RANDOM VARIABLE

In the class of stationary processes a classic example of a locally affine invariant transformation is a module of the Fourier transform $f = |F|$. The examples of affine invariant transformations are also the generalized Hausdorff dimensions. When determining these dimensions the key role is played by the distribution of probability $p(\vec{x_k})$, which is related to the distribution $p(x_k)$ of values $\{x_k\}$. It is precisely its invariance versus the locally affine transformations that implies the same type invariance of the values, which are defined in the terms of this distribution. The information on distribution of probability of value x_k is not sufficient for reconstructing the Hausdorff dimension exactly in the same way as the amplitude spectrum of Fourier transform, and upon neglecting the phase component it s not sufficient for reconstructing the original. In order to analyze these relationships, we narrow our analysis to the class of quasi-periodic signals. By analogy to the periodic signal $x_{k+T} = x_k$, where the phase φ is defined as a parameter from a circle with the length T, having the form of $\varphi \equiv k$ mod$_T$ for d-dimensional quasi-periodic signal being the sum of d periodic signals $x_k^1 + ... + x_k^d$ with incommensurable periods $T_1, ..., T_d$, the phase can be defined as a point $(\varphi_1, ..., \varphi_d)$ on a d-dimensional torus T. The choice of vector $[x_{k+i}, x_{k+2i}, x_{k+3i}, ..., x_{k+mi}] = \vec{x}$ for a sufficiently large m is equivalent to the choice of the signal's phase. It allows for defining the phase space of any signal as a set of points of its Takens' $\mathcal{T}$ space (composed of points $\vec{x}$). In other words, the points of the Takens space contain both the information on the value of the signal (x_k is the first coordinate of point $\mathcal{T}$) as well as on the phase (point $\vec{x_k}$) in which the observed value of the signal is at a given moment. For quasi-periodic signals the full information on the signal is thus contained in the image of mapping of a of a certain d-dimensional torus in the space R^m. This method of reading the dimension d is well known [1] and involves the determination of Hausdorff dimension of a signal [2]. One of the methods of calculating this dimension, based on the observation of the actual signal emitted by an unknown system, involves determination of the set $\mathcal{T}$ through its identification with the probability distribution carrier in the embedding space. Reading the probability distribution $p(\vec{x_k})$ for $\mathcal{T}$ allows one, while relying on well-known methods of calculus of probability, to reconstruct the quasi-periodic signal. The essence of this reconstruction is the assumption that function $k \rightarrow \vec{x_k}$ is a random function with constant distribution on $\mathcal{T}$. The map $\{x_k\} \rightarrow \{p(\vec{x_k})\}$ is obviously invariant with regard to affine transformations. We also prove the opposite theorem saying that each function $f : N \rightarrow Q$, which is invariant with regard to locally affine transformation to N, can be presented as the above assumption $k \rightarrow \{x_k\} \rightarrow \{(\vec{x_k})\} \rightarrow \{p(\vec{x_k})\}$. Thus, the only invariants of affine transformations of signals x_k , and as a result the only values that characterize the quasi-periodic processes, are the functions of parameters of $p(\vec{x_k})$ probability distribution in Takens $\mathcal{T}$ space. Moreover, the changes occurring in a big time-scale in processes modelled by a periodic signal (in a small scale) should be possible to observe in the changes occurring in distribution $p(\vec{x_k})$.

APPLICATIONS

This method applied to vibroacoustic signals allows to detect and trace processes of low energy stages which are unnoticeable or at least unidentifiable by classical Fourier analysis. As we observe in many experiments the structure of histograms approximating the distribution functions of disturbed signal are in many cases more susceptible to changes than Fourier spectrum. Especially the low energy disturbances, poorly visible in Fourier spectrum may be detected as the evolution of the described here random variable and its moments.

References

[1] J.Samsonowicz *Takens space of quasi-periodic signals* Machine Dynamics Problems vol 24
[2] J.Samsonowicz,J.Mączak *Fast algorithm of calculation attractors Hausdorff dimension of time series*, Machine Dynamics Problems vol 24