THE INFLUENCE OF VIBRATION ON THE ONSET OF MARANGONI CONVECTION IN HORIZONTAL FLUID LAYER

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Summary The problem of influence of parametric oscillations on dynamic systems has two particularly interesting cases – the case of high-frequency vibrations and parametric resonance. In the case of high frequency vibration the application of averaging method allowed us to make a conclusion that the vibration smoothes the free boundary if the direction of oscillation is not longitudinal. In the case of finite frequency and transversal influence the continued fractions approach made possible to avoid the conventional restriction to the case of small viscosity and almost-resonant frequencies and effectively calculate the areas of parametric resonances. Our numerical results cover a wide range of the parameters (Marangoni number, viscosity, amplitude and frequency of the oscillation).

PROBLEM FORMULATION

We consider fluid flow in a viscid incompressible homogenous fluid layer, bounded from above by a free surface $x_3 = \xi(x_1, x_2, t)$, and by hard or "soft" (free undeformable) wall from below, which is performing oscillations governed by the law $\tilde{b}\cos\tilde{\omega}t$ along the vector $s = (\cos\varphi, 0, \sin\varphi)$. The governing equations are Oberbek-Boussinesq equations of convective motion in the moving coordinate system.

HIGH-FREQUENCY CASE

The first case studied is the case of high-frequency vibration. The amplitude is considered of order *O(1/ω)*, that is we assume b=*a/ω, a* is of order *O(1)*. The averaging method is applied, as it was done in [1,2]. The unknown variables are represented as the sum of smooth and fast components:

$$\mathrm{v} = \mathrm{v}'(x,t) + \tilde{\mathrm{v}}(x,t,\tau), \quad p = p'(x,t) + \omega\tilde{p}(x,t,\tau), \quad T = T'(x,t) + \frac{1}{\omega}\tilde{T}(x,t,\tau), \quad \xi = \xi'(x,t) + \frac{1}{\omega}\tilde{\xi}(x,t,\tau) \qquad (1)$$

It is possible to express fast components through smooth ones. Substitution of achieved solutions into governing equations leads to appearance of new terms – the vibration-induced mass force (in the case of homogenous fluid they vanish) and vibration-induced stresses [1,2]. The resulting problem can be linearized near quasiequilibrium, resulting in the following eigenvalue problem for the amplitudes of normal perturbations:

$$\begin{gathered}\sigma(D^2-\alpha^2)\mathrm{v} = (D^2-\alpha^2)^2\mathrm{v}, \quad \sigma\Pr\theta = (D^2-\alpha^2)\theta - \mathrm{v}\\ z=0: \mathrm{v} = \sigma\Pr\delta, \quad \mathrm{D}^2\mathrm{v} + \alpha^2\mathrm{v} = Ma\alpha^2(\theta+\delta)\\ (3\alpha^2+\sigma)D\mathrm{v} - D^3\mathrm{v} = \Pr\alpha^2(C\alpha^2 + Ga + \mu_s\alpha th\alpha)\delta \\ D\theta - Bi(\theta+\delta) = 0,\\ z=1: \mathrm{v} = \mathrm{Dv}(\mathrm{D}^2\mathrm{v}) = 0, \quad \mathrm{D}\theta = \mathrm{B}_0\theta = 0\end{gathered} \qquad (2)$$

These equations have the single vibrational parameter $\mu_s = (\mathrm{Re}\sin\varphi)^2/2$, which increases the effective surface tension $C_s = C + Ga/\alpha^2 + \mu_s th\alpha/\alpha$, if the angle of vibration $\varphi \neq 0$. Here $\mathrm{Re} = ah/\nu$ - the vibrational Rayleigh number.
The long-wave asymptotics ($\alpha \to 0$) was also considered. In this case it is possible to represent unknown values as series in powers of α^2 and obtain the terms of asymptotic form of the Marangoni number [1,2].

CASE OF FINITE FREQUENCY

For the same Oberbek-Boussinesq equations the analysis of influence of vibrations of arbitrary frequency and amplitude was made when $\varphi = \pi/2$. For detailed description refer to [3]. The problem has a quasiequilibrium:

$$\mathrm{v}^0 = 0, p^0 = \rho_0 g(t)z, \xi^0 = 0, T^0 = Az + B \qquad (3)$$

The system for normal disturbances can be achieved, by separating variable x, excluding pressure, and introducing flow function ψ:

$$\begin{gathered}(D^2-\alpha^2)\psi_t = (D^2-\alpha^2)^2\psi, \theta_t = \Pr^{-1}(D^2-\alpha^2)\theta - \alpha^2\psi,\\ z=0: \eta_t = \alpha^2\psi, D^2\psi + \alpha^2\psi = \mathrm{Ma}\Pr^{-1}(\theta+\eta),\\ D\psi_t - (D^2-\alpha^2)D\psi + 2\alpha^2 D\psi - (Ga + b\omega^2\cos(\omega t) + C\alpha^2)\eta = 0, D\theta - \mathrm{Bi}(\theta+\eta) = 0,\\ z=h: \psi = 0, D\psi(D^2\psi) = 0, D\theta + B\theta = 0.\end{gathered} \qquad (4)$$

The Floquet solutions of these system are searched as an infinite sum:

$$\psi(z,t) = e^{\sigma t} \sum_{n=-\infty}^{+\infty} \psi_n(z) e^{in\omega t}, \theta(z,t) = e^{\sigma t} \sum_{n=-\infty}^{+\infty} \theta_n(z) e^{in\omega t}, \eta(t) = e^{\sigma t} \sum_{n=-\infty}^{+\infty} \eta_n e^{in\omega t}. \quad (5)$$

Here σ is the Floquet multiplier. Expressing ψ_n, θ_n through c_n we can obtain infinite three-diagonal system for coefficients c_n, which can be written as follows:

$$M_n c_n = -q(c_{n-1} + c_{n+1}), \quad 2q = b\omega^2 \alpha \quad (6)$$

From this system of equations it is possible to derive a dispersion relation in continuous fractions form:

$$-M_0 + \cfrac{-q^2}{-M_1 + \cfrac{-q^2}{-M_2 + \ldots}} = \cfrac{-q^2}{M_{-1} + \cfrac{-q^2}{M_{-2} + \ldots}} \quad (7)$$

For the case of synchronous and subharmonic loss of stability these equations can be reduced to real form. When $\sigma = 0$, that is, the synchronous loss of stability, (7) can be written as:

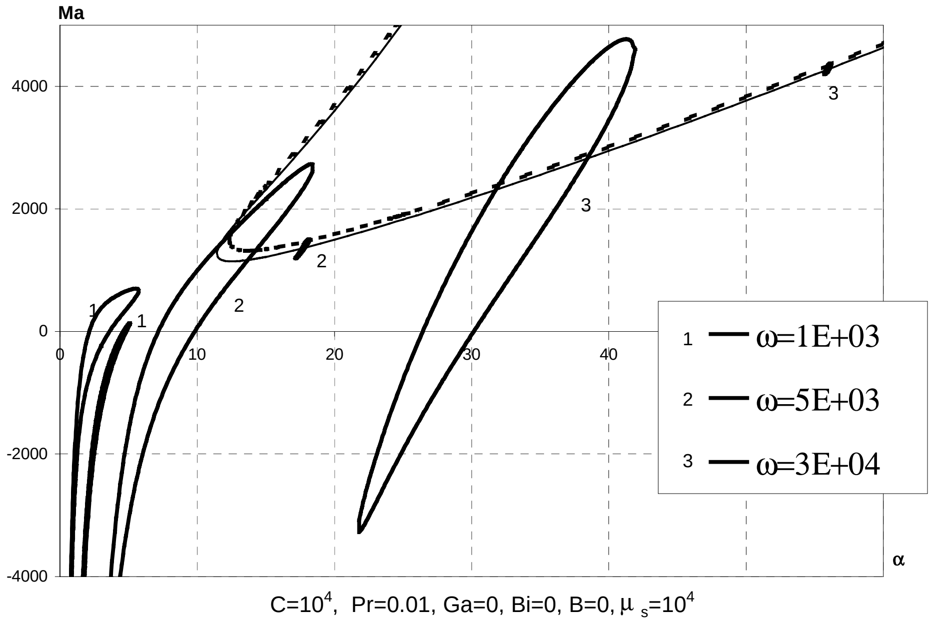
$$\mathrm{Re} \cfrac{q^2}{M_1 - \cfrac{q^2}{M_2 + \ldots}} = \frac{M_0}{2} \quad (8)$$

If $\sigma = i\omega/2$ we get the following equation:

$$\left| M_0 - \cfrac{q^2}{M_1 - \cfrac{q^2}{M_2 - \ldots}} \right|^2 = q^2 \quad (9)$$

These equations allow for a fast and effective numerical stability computation, including the areas of parametric resonance.

NUMERICAL RESULTS



C=10^4, Pr=0.01, Ga=0, Bi=0, B=0, μ_s=10^4

By solving dispersion equations (7)-(9) it is possible to obtain the neutral curves corresponding to different instability types.

One of the most interesting results for the arbitrary frequency computation was the discovery of enclosed areas of parametric resonance and their behaviour. These curves do not disappear with frequency increase, but move up and right along the curve of oscillatory instability, which is illustrated on the figure to the left.

Also, the values of frequency ω were computed, for which the results coincide with high-frequency asymptotics [1,2].

References

[1] Zenkovskaya S.M., Shleykel A.L. Effect of High-Frequency Vibration on the Onset of Convection in a Horizontal Fluid Layer. Doklady Physics, Vol. 47, No. 2, 2002, pp. 148-152.

[2] Zenkovskaya S.M., Shleykel A.L. Influence of High-Frequency Vibration on the Onset of Marangoni Convection in Horizontal Fluid Layer. PMM, 2002, Vol. 66, pp. 573-583.

[3] Zenkovskaya S.M., Shleykel A.L., Novosiadliy V.A. Influence of Vertical Vibrations of Finite Frequency on the Onset of Thermocapillary Convection in Horizontal Layer. Preprint VINITI 27.07.03. N 1440 – 20003. 36 p.