Direct Identification of the State Equation in Complex Nonlinear Systems

by

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SUMMARY

1. INTRODUCTION

A crucial element in the modeling, control and monitoring of adaptive ("smart") systems, is the ability to develop high-fidelity, reduced order and reduced complexity nonlinear (i.e., not-necessarily linear) mathematical models for the physical systems of interest. Recently adaptive estimation approaches have appeared for on-line estimation and identification of hysteretic systems under arbitrary dynamic environments. These adaptive estimation/identification techniques suffer from two drawbacks: (1) they assume that the restoring forces applied to the system's elements are available for measurement and (2) they assume that the differential equation driving these restoring forces can be parameterized as a linear combination of unknown constant parameters and known nonlinear terms.

With the above discussion in mind, the authors have developed an efficient identification algorithm for handling structural systems incorporating severe nonlinearities of the type commonly encountered in the applied mechanics field, spanning the range from large civil structures, to joint-dominated aerospace systems, to micro electromechanical devices (MEMS). The new methodology completely overcomes the above two drawbacks. Specifically, the new approach of this paper solves the problem of estimating/identifying the restoring forces without relying on the need of having the restoring forces directly available for measurement, and without constraining the restoring forces dynamics to have a particular structure.

2. PROBLEM FORMULATION

Building on the basic idea behind the *Restoring Force Method* for the nonparametric identification of nonlinear systems, a general procedure is presented for the direct identification of the state equation of complex nonlinear systems. No information about the system mass is required, and only the applied excitation(s) and resulting acceleration are needed to implement the procedure. Arbitrary nonlinear phenomena spanning the range from polynomial nonlinearities to the noisy Duffing - van der Pol oscillator (involving product-type nonlinearities and multiple excitations) or hysteretic behavior such as the Bouc-Wen model can be handled without difficulty. In the case of polynomial-type nonlinearities, the approach yields virtually exact results. For other types of nonlinearities, the approach yields the optimum (in least-squares sense) representation in nonparametric form of the dominant interaction forces induced by the motion of the system.

To illustrate the basic idea behind the new method, consider a nonlinear single-degree-of-freedom (SDOF) system that is governed by the following differential equation:

$$m\ddot{x} + c_1 x + c_2\dot{x} + c_3 x^3 + c_4 x_1^2\dot{x} + c_5 x f_1(t) + r(t) = c_6 f_2(t) \quad (1)$$

where m is the system mass, the c's are specified system parameters, $f_1(t)$ and $f_2(t)$ external forces acting on m, and $r(t)$ is a hysteretic force of arbitrary type. The nonlinear system characteristics inherent in the system of Eq.(1) covers a broad category of problems: (1) the damped Duffing oscillator under the action of an external exciting force $f_2(t)$, (2) the Noisy Duffing- van der Pol oscillator under the action of disturbances $f_1(t)$ and $f_2(t)$, and (3) a nonlinear system with general hysteretic properties which governs a very broad class of hysteretic systems spanning the range from a completely elastic to a completely elasto-plastic hsyteretic oscillator, specified by parameters $\mathbf{p}$, under exciting force $f_2(t)$.

Rewriting the differential equation Eq.(1) in a form convenient for using conventional numerical solution techniques for initial-value problems, leads to

$$\ddot{x} = g(x, \dot{x}, \mathbf{p}, f_1, f_2) \quad (2)$$

where

$$g = -\frac{c_1}{m}x - \frac{c_2}{m}\dot{x} - \frac{c_3}{m}x^3 - \frac{c_4}{m}x_1^2\dot{x} - \frac{c_5}{m}x f_1(t) - \frac{1}{m}r(t) + \frac{1}{m}f_2(t). \quad (3)$$

The central idea behind the present identification approach is to determine an approximating analytical function $\hat{g}$ that approximates the actual (unknown) system state equation g, with the form of $\hat{g}$ including suitable basis functions that are relevant to the problem at hand. Consequently, for the general nonlinear problem under discussion, a suitable choice of basis would be

$$basis = \{f_1, f_1 d, f_1 v, f_2, f_2 d, PS\} \tag{4}$$

where PS represents the list of basis terms in the power series expansion in the doubly-indexed series

$$PS = \sum_{i=0}^{i_{max}} \sum_{j=0}^{j_{max}} d^i v^j \tag{5}$$

where, for convenience, the symbols d and v are used to represent the system's displacement x and velocity $\dot{x}$, respectively.

3. APPLICATION TO NONLINEAR SYSTEMS

The formulation presented in the paper has been evaluated by calibrating its performance in identifying several types of complex nonlinear systems under deterministic as well as stochastic excitations. The response of an example hysteretic system is shown in Fig.(1). Sample results are shown in Fig.(2), in which a comparison is shown of the simulated hysteretic

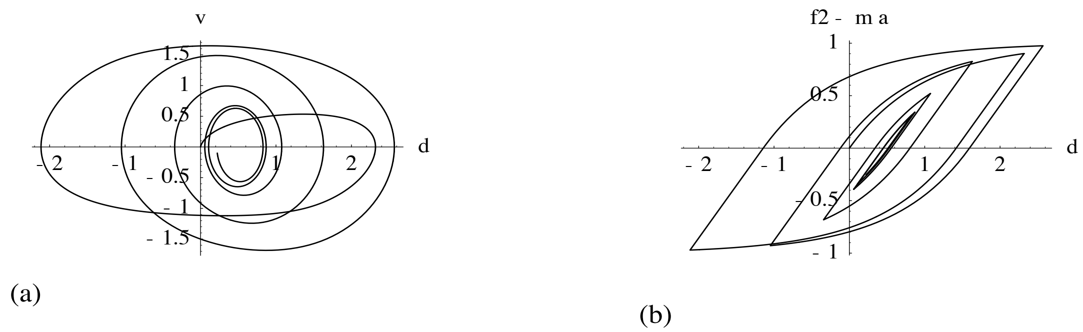

Figure 1: Hysteretic oscillator response data used for identification; (a) Phase diagram of $\dot{x}(t)$ versus $x(t)$; (b) Phase plot of restoring force $f_2(t) - m\ddot{x}(t)$ versus displacement $x(t)$.

system response with a different excitation than what was used for identification, obtained by solving the governing state equation.

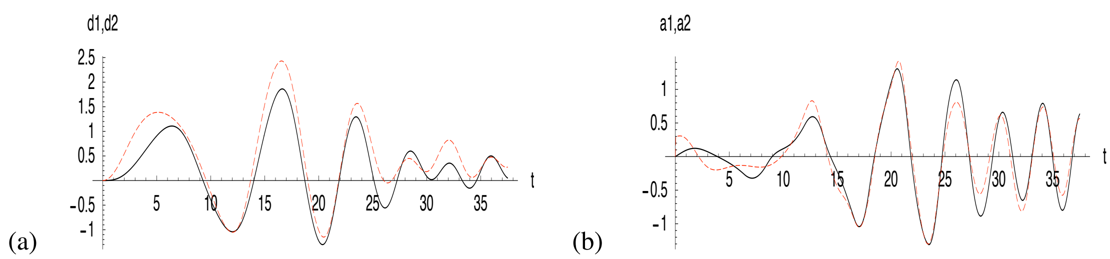

Figure 2: Comparison of simulated hysteretic system response with a different excitation than what was used for identification; (a) Displacement $x(t)$ and $\hat{x}(t)$; (b) Acceleration $\ddot{x}(t)$ and $\hat{\ddot{x}}(t)$.

4. SUMMARY & CONCLUSIONS

A simple yet general approach is presented for the direct identification of the state equation of complex nonlinear systems such as those encountered when dealing with "smart" systems. The procedure is a generalization of the *Restoring Force Method* which develops a low-order, low-complexity, nonparametric representation for a broad class of complex nonlinear systems widely encountered in the applied mechanics field. The utility of the new approach is demonstrated by investigating three different classes of single-degree-of-freedom nonlinear oscillators: the Duffing oscillator, the noisy Duffing - van der Pol oscillator, and the Bouc-Wen hysteretic oscillator. Issues that influence the performance of the method, such as data noise-pollution and the tradeoffs between model fidelity and its complexity, are discussed and guidelines are provided for application to generic cases.