ANALYSIS OF THICK LAMINATED PANEL WITH PIEZOELECTRIC SENSORS BASED ON THREE-DIMENSIONAL THEORY OF ELASTICITY

Daneshmehr A. R. *, Shakeri M. **, Alibiglu A.***

** Scientific Board Member, Islamic Azad University (IAU), central Tehran Branch, Tehran / Iran*
Email: alidaneshmehr@yahoo.com
*** Professor, Dept. of Mechanical Engineering, Amirkabir univ. of Tech. , Tehran / Iran*
**** Assistant Professor, Bu Ali Sina university, Hamedan / Iran*

Summary A study on the elasticity solution of shell panel with piezoelectric sensors is presented. In this paper, the structure is simply-supported, orthotropic and under pressure excitation on outer surface. Three-dimensional equations of equilibrium, which are coupled partial differential equations, are reduced to ordinary differential equations with variable coefficients by means of Fourier series expansion in circumferential and axial directions. The resulting ordinary differential equations are solved by Galerkin finite element method. Numerical results are presented for [0/90/P] lamination.

INTRODUCTION

In recent years, smart structures with piezoelectric sensors and actuators have attracted serious attention for they can sense and alter the mechanical response during in-service operation. On the other hand, light-weight shell and panel structures may be one of the most popularly used structures in space vehicles. For these reasons, shell-type smart structures have become the subject of focus for many researchers. Tzou and his co-workers published a series of papers on finite element and analytical analysis of piezoelectric shell-type continua (Tzou , 1993 and Tzou et al., 1994). In addition to the aforementioned approximate analysis, exact studies of piezoelectric circular cylindrical shells (Chen et al., 1996) and piezothermoelastic shells (Chen and Shen, 1996a) were also performed. The theories of plates and shells coupled with piezoelectricity theory were applied to piezoelectric sensor and actuator design (Lee 1996). The piezoelastic solutions to long cylindrical panel and shell structures were also presented (Kapuria 1997) . Elastic solution of orthotropic thick laminated cylindrical panels subjected to dynamic loading was obtained by author (Shakeri 2002).

The objective of the present study is to investigate the elastic solution of cross-ply laminated panels with piezoelectric layer using three-dimensional theory of elasticity.

GOVERNING DIFFERENTIAL EQUATIONS

The equations of equilibrium in the absence of body force and the charge equation of equilibrium of electrostatics in cylindrical coordinates are:

$$\begin{aligned}
&\frac{\partial\sigma_r}{\partial r}+\frac{\sigma_r-\sigma_\theta}{r}+\frac{1}{r}\frac{\partial\tau_{r\theta}}{\partial\theta}+\frac{\partial\tau_{zr}}{\partial z}=0\\
&\frac{\partial\tau_{r\theta}}{\partial r}+\frac{1}{r}\frac{\partial\sigma_\theta}{\partial\theta}+\frac{\partial\tau_{z\theta}}{\partial z}+2\frac{\tau_{r\theta}}{r}=0\\
&\frac{\partial\tau_{rz}}{\partial r}+\frac{\partial\tau_{z\theta}}{r\partial\theta}+\frac{\partial\sigma_z}{\partial}+\frac{1}{r}\tau_{rz}=0\\
&\frac{\partial D_r}{\partial r}+\frac{D_r}{r}+\frac{\partial D_\theta}{r\partial\theta}+\frac{\partial D_z}{\partial z}=0
\end{aligned} \qquad (1)$$

SOLUTION OF GOVERNING DIFFERENTIAL EQUATIONS

The solution satisfying the boundary conditions maybe assumed as:

$$u_r=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\phi_r(r)\sin(b_m\theta)\sin(b_n z) \qquad u_\theta=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\phi_\theta(r)\cos(b_m\theta)\sin(b_n z)$$

$$u_z = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\phi_z(r)\sin(b_m\theta)\cos(b_n z) \qquad \psi = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\psi(r)\sin(b_m\theta)\sin(b_n z) \qquad (2)$$

NUMERICAL RESULTS AND DISCUSION

A sensor consisting of a three-layered cross-ply panel which its sequence lay-up is (0/90/P) composed of graphite-epoxy and piezoelectric is considered. The in-plane shear stress and radial normal stresses across the thickness are as the figures 1 and 2.

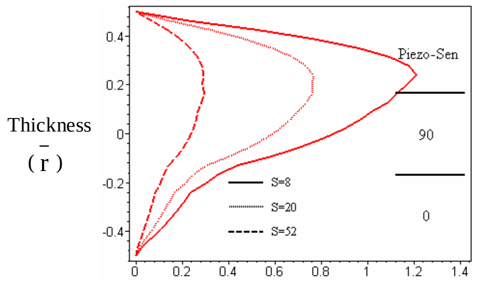

Fig.1 In plane shearing stress ($\tau_{r\theta} / P_0$)

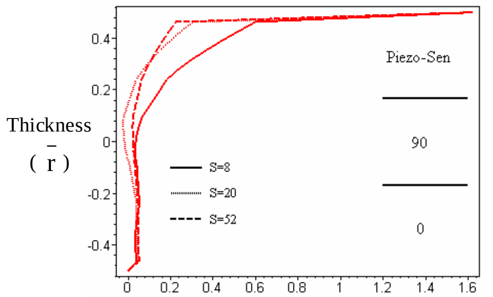

Fig.2 Radial stress (σ_r / P_0)

CONCLUSION

A study on the three-dimensional elasticity solution of shell panel piezoelectric sensors is presented. In this paper, the structure is simply-supported, orthotropic and under pressure excitation. The present study has shown that the Fourier series expansion method is suitable for the mechanical displacement and electric potential analysis in panel-type sensors.

The piezoelectric effect in the piezoelectric layer subjected to outer pressure is investigated in detail. In analytical investigation of the piezoelectric structures, it is always assumed that the electric potential in the piezoelectric layers varies linearly and the displacements change in the form of prescribed functions across its thickness [10]. However, it has been shown that the distributions of the mechanical displacements and electric potential of piezoelectric response are very complicated and cannot be treated as pure elastic structures or piezoelectric structures. Therefore, three-dimensional analysis of piezoelastic behavior of structures is recommended even for thick and thin laminated structures. Since a comprehensive and exact study of active piezoelectric structures is still unavailable, the present work provides an enhanced insight to the mechanical and electric behaviors of this type of smart structure.

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