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INSTABILITY OF THE FAR WAKE BEHIND A WIND TURBINE

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Summary A stability analysis of the far wake behind a N-bladed wind turbine has been carried out. In conflict with experimental data, previous studies based on stability analysis of point vortices predicted instability at all operating conditions. In the present study the wake description is based on a (N+1)-vortex model consisting of N helical tip vortices and a hub vortex, exposed to a constant axial velocity field. As a generalization, the stability problem is reduced to the task of finding a procedure for the evolution of the velocity induced by a helical vortex system. An algebraic solution is obtained that provides for an efficient analysis of motion and stability of the (N+1) vortex systems covering a range of helical pitch variations. The stability properties depend basically on vortex strength, helical pitch and vortex core radius. To relate these properties to the operating conditions of a wind turbine, they are approximately expressed in terms of thrust coefficient and tip speed ratio. In accordance with experiments, the far wake is shown to consist of both stable and unstable flow regimes.

Modern wind turbines are often grouped in parks or “wind farms” in order to provide power to an electrical grid from a large number of turbines at the same site. At some wind conditions, however, one or more turbines may stand in the wakes of other turbines. This causes the loading be dominated by strong periodically varying forces that increase the fatigue loads and hence reduce the lifetime of the turbine. As seen in Fig. 1a the flow behind a wind turbine is dominated by a system of strong trailing tip vortices that forms the dynamics of the far wake. In some cases the vortices become unstable and breaks down, as seen in Fig. 1b. It is clear that if a wind turbine is located in a wake consisting of stable tip vortices, the fatigue loading is more severe that if the vortices break down into a continuous vortex sheet. It is of interest study under which conditions the vortices break down. The present work is focused on studying far-wake features of wind turbines by analyzing stability properties of systems of helical vortices.

Fig. 1.Visualisations of far wake: (a) stable regime [4] and (b) unstable one with the wake decay (data by RISO, Denmark).

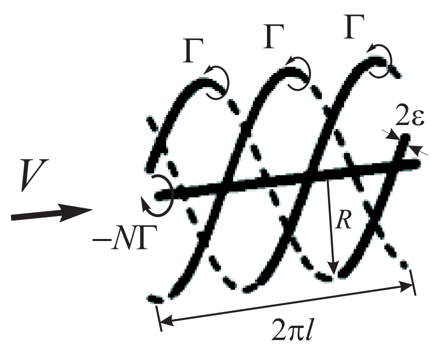


Fig. 2. Scheme of the (N+1) helical vortex structure.

At a sufficiently large distance in the wake behind a wind turbine the vorticity is concentrated in N helical tip vortices that are located on a cylindrical surface of radius R and equally spaced $2\pi/N$ radians apart from each other, where N is the number of blades. Each of the tip vortices has strength Γ and a vortex core radius ε. Further, a hub vortex with strength ($-N\Gamma$) is lying along the axis of rotation. The stability problem is thus characterized by a system of ($N + 1$) vortices that are exposed to a constant axial wind field of speed V (see Fig. 2). This ($N + 1$)-vortex system, with constant pitch, $2\pi l$ (or $\tau = l/R$ in dimensionless form) is the simplest possible description of a wake, e.g. Joukowski [1]. Since the axial flow behind a wind turbine has a “wake-like” profile, we consider a system of left-handed helical vortices [2]. Representing the ($N + 1$)-vortex system by point vortices or straight vortex filaments (the limiting case of a helical vortex with infinite pitch) the vortex system (with hub vortex) is absolutely unstable (see, e.g. [3]). This result, obtained as the simplest special case ($\tau = \infty$), is not in agreement with the findings of visualizations of wind turbine wakes, e.g. [4].
In order to study instabilities of the ($N + 1$) helical vortex system, one turns to the helical variables (r, χ) with corresponding velocity projections (u_r, $u_\chi = u_\theta + u_z/\tau$). In this context, the problem of stability of the N-vortex configuration without hub vortex was reduced to a two-dimensional case [5, 6]. In the present work the stability analysis is further extended to the general problem of ($N + 1$) helical vortices with an additional axial translation of the vortex system. To apply the present model to analyze far-wake stability, we need to represent the geometrical and kinematical parameters of the wake model (τ, V and Γ) via typical wind turbine operating characteristics (axial interference coefficient a, and tip speed ratio λ). The

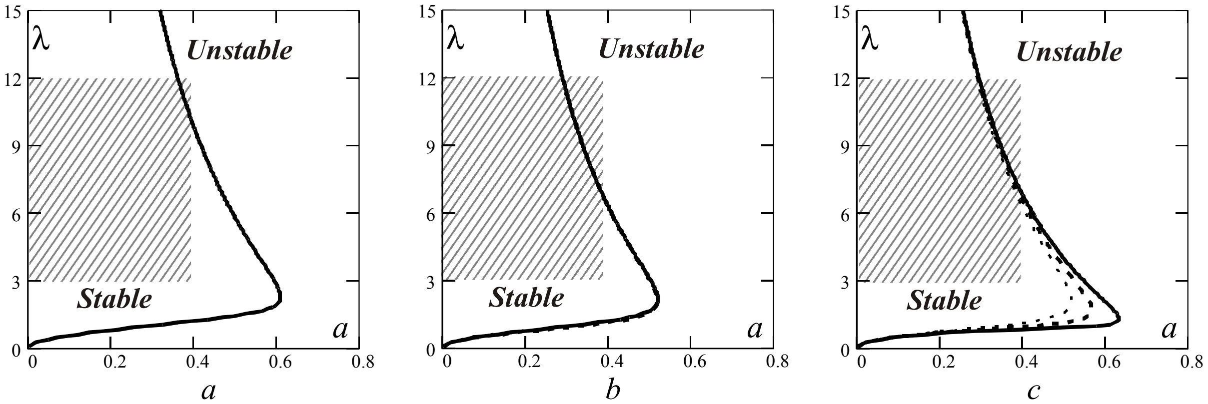
parameter a is defined as the ratio between the axial velocity induced by the helical vortices in the wake and the wind speed. If we neglect of the vortex core size, after averaging axial velocity from [6], we get $a = N\Gamma/2\pi l u_0 = G/\tau$, where $G = N\Gamma/2\pi RV$ is the dimensionless total circulation of the tip vortices. As a next simplification, we present $G = 2a(1-a)/\lambda$ from lifting disk theory. As a result, we get $\tau = 2(1-a)/\lambda$. To replace V we use the defintion of G, i.e. $N/G = 2\pi RV/\Gamma = N\lambda/2a(1-a)$. In terms of these parameters the modified correlation equation from [6] reads

$$\frac{1}{\vartheta(a,\lambda)}\left(2(1-a)+\frac{0.5\lambda^2}{1-a}\left(\ln\left(\frac{2(1-a)}{N\varepsilon\vartheta(a,\lambda)}\right)+\frac{7}{4}\right)\right)+\frac{2(1-a)\lambda^2}{\vartheta^3(a,\lambda)}\left(\ln(N)-\left(1-\frac{\lambda^2}{16(1-a)}\right)\left(\frac{N}{m}-C-\psi\left(-\frac{m}{N}\right)\right)\right)+$$

$$\frac{(1-a)\lambda^6}{\vartheta^7(a,\lambda)}\left(32\left(\frac{\varsigma(3)}{N^2}-1\right)\frac{(1-a)^4}{\lambda^5}-24\left(\frac{\varsigma(3)}{N^2}+1\right)\frac{(1-a)^2}{\lambda^3}+\left(\frac{3\varsigma(3)}{4N^2}-6\right)-\frac{0.5\lambda^2}{(1-a)^2}\right)+\frac{(N-0.5)a-N}{2a(1-a)^2}\lambda^2+$$

$$\frac{(1-a)\lambda^6\left(4(1-a)^2-\lambda^2\right)}{\vartheta^9(a,\lambda)}\left(32\frac{(1-a)^4}{\lambda^4}-24\frac{(1-a)^2}{\lambda^2}+\frac{3}{4}\right)\frac{1-N^2}{N^2}\varsigma(3)+m(N-m)\frac{\vartheta^3(a,\lambda)}{8(1-a)^3}+1=0$$

where $\vartheta(a,\lambda)=\sqrt{4\left(1-a^2\right)+\lambda^2}$; C = 0.577215… is the Euler constant. The psi-function $\psi(\cdot)$ for several typical arguments is ψ(-1/2) = 0.03949…, ψ(-1/3) = 1.68177…, ψ(-2/3) = -1.63203…, ψ(-1/4) = 2.91414…, and ψ(-3/4) = -2.89412…



a *b* *c*

Fig. 3. Diagram of possible operating regimes of wind turbines (crosshatched region) and neutral stability curves for the far wake (a) N = 2; (b) N = 3; (c) influence of vortex core radius - ε on the neutral stability curves for N = 3: ε = 0.1 – solid line, ε = 0.01 – dashed line and ε = 0.001– dotted line.

Figure 3 shows neutral stability curves for the (N + 1) – vortex system model of the far wake as a function of the number of blades, i.e. 2 or 3 blades. Typical wind turbine operating regimes (crosshatched region in Figure 3) are seen to lie mainly in stable domains. However, in some cases wind turbines operate under conditions in which the vortex structure in the wake becomes unstable, in agreement with observations in [7]. Investigation of the influence of the vortex core radius is presented in figure 3*c*. The influence is seen to be small and it may be ignored.

CONCLUSIONS

In conclusion, for the first time an analytical study has of the stability problem of a (N + 1) - vortex system that models the far wake behind a wind turbine has been carried out. Typical operating regimes of wind turbines have been compared to stability characteristics and it was found that, depending on number of blades and operating conditions, both stable and unstable vortical wake structures appear. The analytical form of the correlation equation represents the first step in understanding and controlling wind turbines in parks.

References

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