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WRINKLING AND BUCKLING OF ISOTROPIC BIOLOGICAL TISSUES

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Summary The problem of the wrinkling and buckling of membranes characterized by an isotropic Tong-Fung type constitutive behavior has been formulated and solved within the framework of finite strain hyperelasticity. The formulation has been guided by the theories proposed by Wu and Canfield and Pipkin [1,2]. A criterion for the wrinkling based on the *natural width*, which defines the natural contraction of a membrane loaded in uniaxial tension, has been introduced. The out of plane geometric nonlinearities have been treated as constitutive nonlinearities through a modification of the elastic potential. Close form solutions have been found for the natural width and the modified elastic potential. The model has been implemented in a finite element code and applied to simulate procedures of reconstructive surgery where the extrusion of the wound edges may occur after the suture due to the buckling and wrinkling of the skin.

INTRODUCTION

The skin is a complex biological tissue that under short-term loading displays a highly nonlinear and pseudo-elastic mechanical response and finite strains. At low strains the response is mainly controlled by the elastin and at large strains by the collagen fibers, after they have become uncrimpled. In addition, in its natural state the skin is stretched and subject to a state of biaxial tension.

The response of the skin under short-term loading is well described by the phenomenological model proposed by Tong and Fung [3]. The model defines the elastic potential $\hat{W}(\mathbf{E})$, or elastic strain energy per unit surface area, as the sum of a quadratic and an exponential function of the Green-Lagrange membrane strain tensor, **E**, with components $\varepsilon_{\alpha\beta}$ (α, β = 1,2). Under the assumption of isotropy, which well describes the skin at certain anatomical locations such as the scalp, and assuming the strains to be large enough so that the quadratic part of $\hat{W}$ becomes negligible, the model is:

$$\hat{W}(\mathbf{E}) = c\ \exp\left\{B\left[\varepsilon_{11}^2+\varepsilon_{22}^2+2\beta\varepsilon_{11}\varepsilon_{22}+2(1-\beta)(\varepsilon_{12}\varepsilon_{21})\right]\right\}, \tag{1}$$

where c, B and β are constants to be determined experimentally. The second Piola-Kirchhoff membrane stress tensor, PK2, is derived from (1) and its components are $\sigma_{\alpha\beta} = \partial\hat{W}/\partial\varepsilon_{\alpha\beta}$ (α, β = 1,2). In [4] the authors calibrated the model (1) using *in vivo* tests on human scalp skin in order to describe the constitutive response of skin flaps within procedures of reconstructive and cosmetic surgery. The investigation also led to the determination of the principal stretch, λ_n, that defines the isotropic and homogeneous stress field of the scalp skin in its natural state. The inferred parameters, c = 1.13 kPa cm, B = 0.89, β = 0.6 and λ_n = 1.1, have been used to analyze the membranes shown in Figs. 1 and 2.

The model (1) does not describe the response of the skin in compression: like all elastic membranes, the skin has a negligible flexural stiffness and it buckles or wrinkles in the presence of strains that would require compressive stresses. Three distinct regions of behaviour may be identified in a stretched membrane (Fig. 1): taut regions, T, where the skin is in biaxial tension and the model (1) is applicable; buckled regions, B, where the skin has buckled and is inactive and wrinkled regions, W, where the skin develops a continuous distribution of infinitesimal wrinkles and is in a state of uniaxial tension in the direction of the wrinkles. Within procedures of reconstructive surgery the phenomena of wrinkling and buckling may lead to the extrusion of the edges of the wound after suture or *dog-ear* formation.

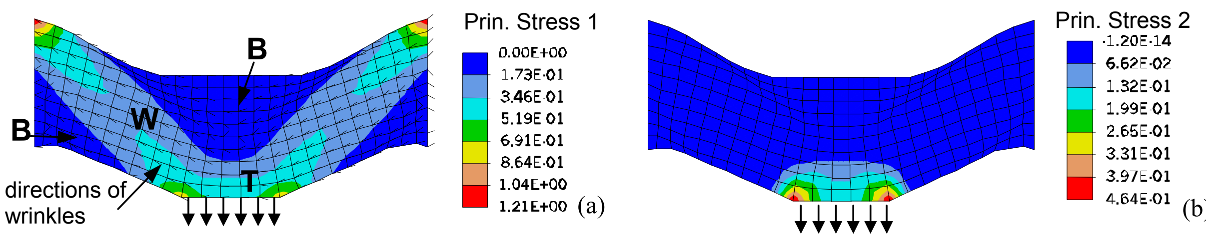

Figure 1: Principal membrane stress fields (PK2, in [kPa cm]) plotted on the actual deformed configuration of a rectangular membrane (reference configuration: 75×25×1mm) laterally clamped and subject to uniform vertical displacements v=10 mm as shown. Analysis performed using model (2) with elastic constants derived in [4]. Buckled regions, B, are inactive (blu in (a) and (b)), wrinkled regions, W, have only one non-zero principal stress (light blu in (a), blu in (b)) and taut regions, T, are in biaxial tension. Direction of the wrinkles defined by the principal stress direction (shown in (a) superposed onto FE mesh). The response is similar to that of polyethylene membrane tested in [5] and subject to same loading configuration (see Fig.1 of [5]).

MODEL, FINITE ELEMENT IMPLEMENTATION, SIMULATION OF RECONSTRUCTIVE SURGERY

In this paper the problem of the wrinkling and buckling of isotropic and hyperelastic membranes characterized by a Tong-Fung constitutive behaviour is formulated within the framework of finite strain elasticity following the approaches of Wu and Canfield and Pipkin [1,2]. It is assumed that the wrinkling occurs in the direction of the principal strains and that the directions of the principal Cauchy stresses do not change because of the wrinkling.

The membrane *natural width*, w, is first defined as the natural transverse contraction of a membrane in uniaxial tension. For a uniaxial tension σ_I the natural contraction corresponds to the minimum of $\hat{W}$ in Eq. (1) with respect to

ε_{II} and is given by $\varepsilon_{II} = w(\varepsilon_I) = -\beta\varepsilon_I$. The natural width depends on the constant β, which is the parameter that at incipient deformation can be assimilated to a generalized Poisson coefficient, $\beta = (\partial\sigma_{11}/\partial\varepsilon_{22})(\partial\sigma_{11}/\partial\varepsilon_{11})^{-1}$.

A criterion for the wrinkling/buckling of the membrane is set up in terms of w and the principal strain components, ε_I and ε_{II}, and a *relaxed energy density* is introduced through a modification of $\hat{W}$ to describe the actual stress fields:

	Criterion for wrinkling/buckling	Relaxed energy density	
Taut regions:	if $\varepsilon_I \geq -\beta\varepsilon_{II}$ and $\varepsilon_{II} \geq -\beta\varepsilon_I$	$\hat{W}_{rel}(\varepsilon_I,\varepsilon_{II}) = \hat{W}(\mathbf{E}) = c\,\exp\left\{B\left[\varepsilon_I^2+\varepsilon_{II}^2+2\beta\varepsilon_I\varepsilon_{II}\right]\right\}$	
Wrinkled regions:	if $\varepsilon_I \geq 0$ and $\varepsilon_{II} < -\beta\varepsilon_I$	$\hat{W}_{rel}(\varepsilon_I) = c\,\exp\left\{B\left[\left(1-\beta^2\right)\varepsilon_I^2\right]\right\}$	(2)
	if $\varepsilon_{II} \geq 0$ and $\varepsilon_I < -\beta\varepsilon_{II}$	$\hat{W}_{rel}(\varepsilon_{II}) = c\,\exp\left\{B\left[\left(1-\beta^2\right)\varepsilon_{II}^2\right]\right\}$	
Bukled regions	if $\varepsilon_I < 0$ and $\varepsilon_{II} < 0$	$\hat{W}_{rel}(\varepsilon_I,\varepsilon_{II}) = 0$	

The expression of the relaxed energy density in the wrinkled regions has been obtained by imposing $\hat{W}_{rel}(\varepsilon_I) = \hat{W}(\varepsilon_I, \varepsilon_{II} = w(\varepsilon_I))$ for $\varepsilon_I > 0$ [2]. Considering a wrinkled region where $\varepsilon_I > 0$, the stress state is uniaxial and the PK2 principal membrane stresses are $\sigma_I = 2cB\left(1-\beta^2\right)\varepsilon_I \exp\left\{B\left[\left(1-\beta^2\right)\varepsilon_I^2\right]\right\}$ and $\sigma_{II} = 0$.

The model (2) has been implemented in the finite element code FEAP. The simulation of the stretching of a membrane shown in Fig. 1 highlights the potency of the numerical model to predict the distinct regions of behavior. Figure 2 is the result of a preliminary application of the model (2) to simulate reconstructive surgery. The figure shows membrane stress fields after the suture of a circular excision on a skin flap that has been previously undermined. Solutions obtained assuming that the skin is able to transfer compressive stresses and assuming that the skin wrinkles and buckles in compression are compared to highlight differences in the post-suture response. The extrusion of the edges of the wound takes place in the region ahead of the excision along the line of the incision.

CONCLUSIONS

The problem of the wrinkling and buckling of biological membranes characterized by a isotropic Tong-Fung type constitutive behavior has been solved within the framework of finite strain hyperelasticity. The natural width, which is used in the criterion for the wrinkling and in the definition of the modified elastic potential, is found to depend on only one of the model parameters. The implementation of the model in the finite element code FEAP enables the simulation of procedures of reconstructive surgery where wrinkling and buckling of the skin may occur after the suture of the edges of the wound.

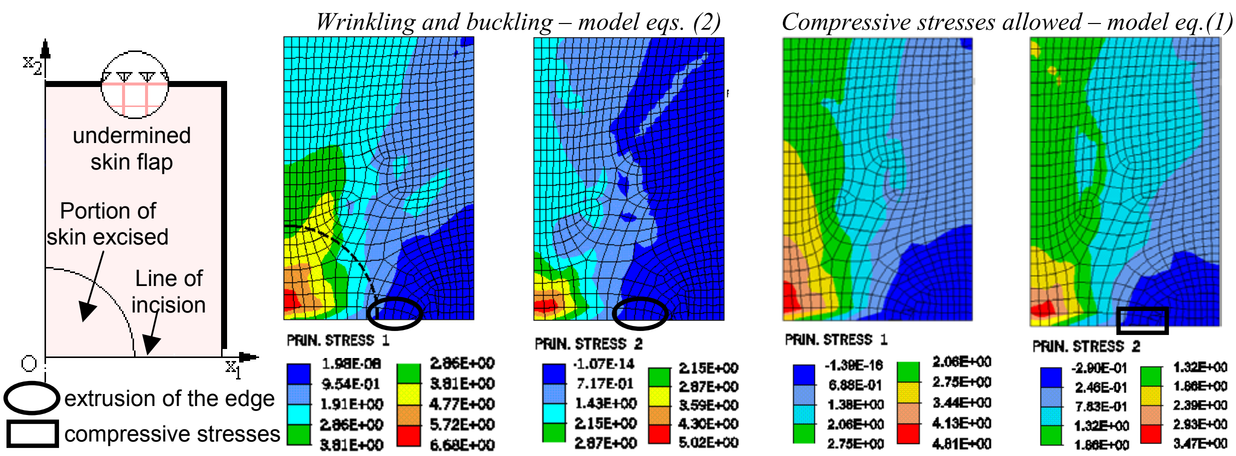

Figure 1: Principal stress fields (PK2, in [kPa cm]) after the excision and the suture of a circular portion of the skin (radius 27 mm) in an undermined skin flap similar to those tested and analyzed in [4] (dimensions of the flap in the natural configuration : 60×90×1 mm).

References

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