ON THE DAMPING OF A PIEZOELECTRIC TRUSS

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Abstract This paper re-examines the classical problem of active and passive damping of a piezoelectric truss. The active damping strategy is the so-called IFF (Integral Force Feedback) which has guaranteed stability; both voltage control and charge (current) control implementations are examined; they are compared to resistive shunting. It is shown that in all three cases, the closed-loop eigenvalues follow a root-locus; closed form analytical formulas are given for the poles and zeros and the maximum modal damping. It is shown that the performances are controlled by two parameters: the modal fraction of strain energy ν_i in the active strut and the electromechanical coupling coefficient k. The paper also briefly addresses the inductive shunting, for which a new parameter comes in: the tuning ratio ω_e/ω_i between the electrical circuit and the mechanical vibration. Due to space limitations, this paper includes only a small part of the sectional lecture at the 21st ICTAM.

1. Introduction

The active damping of a truss with piezoelectric struts has been largely motivated by producing large, lightweight spacecrafts with improved dynamic stability; this classical problem has received a lot of attention over the past 15 years and very effective solutions have been proposed (e.g.[1]). One of them known as Integral Force Feedback (IFF) is based on a collocated force sensor and has guaranteed stability [2].

Traditionally, the piezoelectric actuators have been controlled with a voltage amplifier; this is known to lead to substantial hysteresis caused by the ferroelectric behavior of the material, which requires an external sensor and closed-loop control for precision engineering applications. On the contrary, charge control allows to achieve a nearly linear relationship between the driving electrical value and the free actuator extension (e.g.

[3]). One of the purposes of this paper is to re-examine the theory of the IFF when it is implemented with a current amplifier (charge control).

For space applications, because of the inherent constraints of the launch loads, the space environment, and the impossibility of in-orbit maintenance, there is a strong motivation to reduce or eliminate the power electronics associated to the piezoelectric actuators as well as the complex electronics associated to sensing (particularly in the sub-micron range where the sensor sensitivity becomes an issue). This has motivated the use of passive electrical networks as damping mechanisms [4], [5], [6]. The efficiency of such a damping mechanism depends very much on the ability to transform mechanical (strain) energy into electrical energy, measured by the electromechanical coupling factor, and recent improvements have led to piezoelectric materials with coupling factors of $k_{33} = 0.7$ and more ($0.9 \sim 0.95$ is advertised for PMN-PT single cristal), making them a very attractive option for damping trusses. The second purpose of this paper is to compare the passive and the active options. They are presented in a very similar formalism and closed-loop results are presented, which allow a direct evaluation of the performances in terms of two physical parameters: the *modal fraction of strain energy* ν_i in the piezoelectric strut and the *electromechanical coupling coefficient* k. Due to space limitations, the discussion is restricted to SISO systems, but most of the extensions for decentralized MIMO control apply here (some of them are discussed in [2]).

Consider the constitutive equations of a one-dimensional piezoelectric material:

$$\left\{ \begin{array}{c} D \\ S \end{array} \right\} = \left[\begin{array}{cc} \varepsilon^T & d_{33} \\ d_{33} & s^E \end{array} \right] \left\{ \begin{array}{c} E \\ T \end{array} \right\}, \tag{1}$$

where the standard IEEE notations have been used. If one assumes constant strain, stress and electric fields over the actuator, the constitutive equation can be integrated over the volume of the actuator. With the notations of Fig. 1, one gets

$$\left\{ \begin{array}{c} Q \\ \Delta \end{array} \right\} = \left[\begin{array}{cc} C & nd_{33} \\ nd_{33} & 1/K_a \end{array} \right] \left\{ \begin{array}{c} V \\ f \end{array} \right\}, \tag{2}$$

where $Q = DAn$ is the total electric charge, $\Delta = Sl$ is the total extension, $f = AT$ is the total force and V is the total voltage applied to the piezo ($E = nV/l$). In Eq. (2), $C = \varepsilon^T An^2/l$ is the capacitance of the piezo, $K_a = A/(s^E l)$ is the stiffness of the piezoelectric strut under short-circuited conditions ($V = 0$). Alternatively, using the current

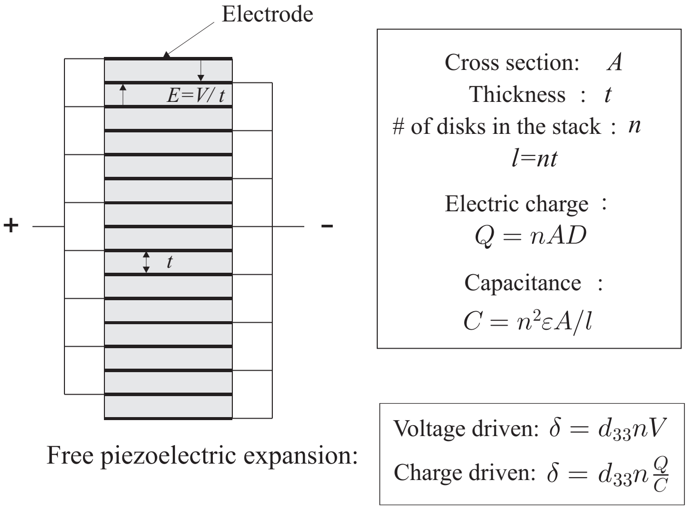

Figure 1. Piezoelectric linear actuator.

I instead of Q, Eq. (2) can be written

$$\left\{ \begin{array}{c} I \\ \Delta \end{array} \right\} = \left[\begin{array}{cc} sC & snd_{33} \\ nd_{33} & 1/K_a \end{array} \right] \left\{ \begin{array}{c} V \\ f \end{array} \right\}, \tag{3}$$

where s is the Laplace variable.

Consider the truss structure of Fig. 2, provided with an active strut consisting of a piezoelectric linear actuator co-linear with a force sensor; the total force in the strut is f. Assuming that the system is undamped, the dynamics of the truss is governed by

$$M\ddot{x} + K^* x = bf, \tag{4}$$

where K^* is the stiffness matrix with the active strut removed (Fig. 2) and b is the influence vector of the active strut in the global coordinate system (the non-zero components of b are the direction cosines of the active strut in the structure). To make things simpler, but without loss of generality, we will assume that the active strut is massless, so that the mass matrix M is the same, with and without the active strut. Note that the total extension of the active strut can be expressed in terms of the global structural displacements as

$$\Delta = b^T x, \tag{5}$$

where b^T is the transposed of the influence vector appearing in the previous equation.

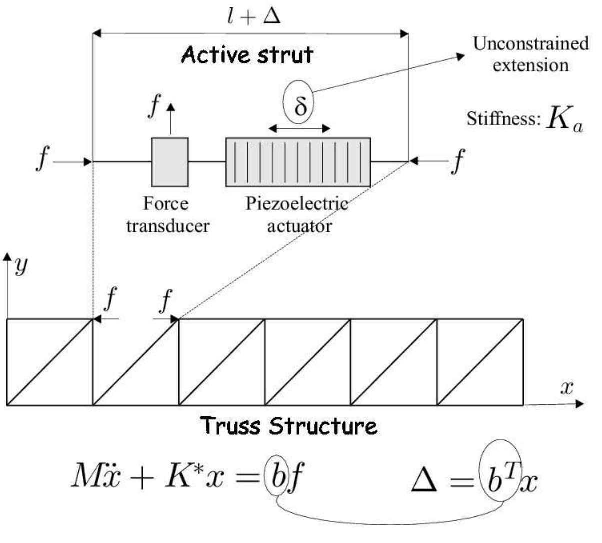

Figure 2. Truss equipped with an active strut (piezoelectric linear actuator co-linear with a force sensor).

2. Voltage Control

Combining Eqs. (3)-(5), one easily gets

$$M\ddot{x} + (K^{*} + K_a bb^{T})x = bK_a d_{33}\,\mathrm{nV}, \tag{6}$$

where $\delta = d_{33}nV$ is the unconstrained expansion under voltage V. In Laplace form, the equation can be written alternatively

$$Ms^2x + (K^{*} + K_a bb^{T})x = bK_a\delta. \tag{7}$$

From Eq. (3), the output equation is

$$y = f = K_a(b^{T}x - \delta). \tag{8}$$

According to the IFF, the collocated force output is integrated and fed back to the voltage actuator:

$$\delta = \frac{g}{K_a s}y. \tag{9}$$

Combining Eqs. (7)-(9), one gets the closed-loop characteristic equation

$$[Ms^2 + (K^{*} + K_a bb^{T}) - \frac{g}{s+g}(K_a bb^{T})]x = 0. \tag{10}$$

The asymptotic roots for $g \to 0$ are solutions of

$$[Ms^2 + (K^{*} + K_a bb^{T})]x = 0, \tag{11}$$

which corresponds to the global truss when the electrodes of the active strut are short-circuited, while for $g \to \infty$ (open-loop zeros), the eigenvalue problem is reduced to

$$[Ms^2 + K^*]x = 0, \tag{12}$$

which corresponds to the situation where the axial contribution of the active strut has been removed.

3. Charge Control

Upon inverting Eq. (3) and using the fact that

$$k^2 = d_{33}^2/(s^E \varepsilon^T) = n^2 d_{33}^2 K_a/C,$$

one gets

$$\left\{ \begin{array}{c} V \\ f \end{array} \right\} = \frac{K_a}{sC(1-k^2)} \left[\begin{array}{cc} 1/K_a & -snd_{33} \\ -nd_{33} & sC \end{array} \right] \left\{ \begin{array}{c} I \\ \Delta \end{array} \right\}. \tag{13}$$

Combining the second of these equations with Eq. (4), one gets

$$M\ddot{x} + \left[K^* + bb^T \frac{K_a}{1-k^2} \right] x = b \left[\frac{K_a}{1-k^2} \right] \cdot d_{33} n \frac{I}{sC}. \tag{14}$$

From the second Eq. (13), we note that $d_{33}nI/(sC)$ is the unconstrained expansion ($f = 0$) under the electric charge $Q = I/s$, that we will denote again by δ. As compared to Eq. (7) of the previous section, Eq. (14) shows that the piezoelectric strut behaves with an increased stiffness $K_a/(1-k^2)$, which is the strut stiffness under open electrodes conditions ($I = 0$). Also from Eq. (13), the output equation reads:

$$y = f = \frac{K_a}{1-k^2}(b^T x - \delta). \tag{15}$$

As in the previous section, we introduce the IFF feedback law

$$\delta = \frac{(1-k^2)g}{K_a s} y, \tag{16}$$

(same as Eq. (9), except for the stiffness of the piezoelectric strut). Combining Eqs. (14)-(16), one gets the closed-loop characteristic equation:

$$\left[Ms^2 + \left(K^* + \frac{K_a}{1-k^2} bb^T \right) - \frac{g}{s+g} \frac{K_a}{1-k^2} bb^T \right] x = 0, \tag{17}$$

identical to Eq. (10) except that the open electrode stiffness $K_a/(1-k^2)$ has been substituted to the short-circuited stiffness K_a. The asymptotic roots for $g \to 0$ (open-loop poles) are solutions of

$$\left[Ms^2 + \left(K^* + \frac{K_a}{1-k^2} bb^T\right)\right] x = 0, \tag{18}$$

which corresponds to the global truss when the electrodes of the active strut are open, while for $g \to \infty$, the eigenvalue problem is reduced to

$$[Ms^2 + K^*]x = 0, \tag{19}$$

which corresponds to the situation where the active strut has been removed. Thus, the open-loop zeros are identical to those for the voltage control case.

4. Passive Shunting

Figure 3(a) shows the electrical analog of the piezo. If a passive shunt of admittance Y_{SH} is connected in parallel with the piezo (Fig. 3(b)), the constitutive equations of the active strut become

$$\left\{ \begin{array}{c} I \\ \Delta \end{array} \right\} = \left[\begin{array}{cc} sC + Y_{SH} & snd_{33} \\ nd_{33} & 1/K_a \end{array} \right] \left\{ \begin{array}{c} V \\ f \end{array} \right\}. \tag{20}$$

This equation applies to active as well as passive control. In the former case, the control input is I or V; in the latter case, $I = 0$ and V can be eliminated from Eq. (20). Combining with Eq. (4) and (5), one finds:

$$\left[Ms^2 + \left(K^* + K_a bb^T\right) + \frac{k^2 K_a bb^T}{(1-k^2) + Y_{SH}/sC}\right] x = 0. \tag{21}$$

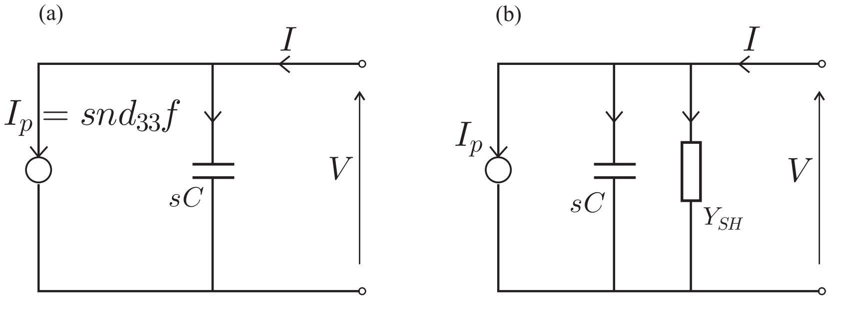

Figure 3. (a) Electrical analog of the piezo. (b) with a passive shunt.

Note that we recover the expected asymptotic forms for $Y_{SH} = 0$ (open electrodes)

$$\left[Ms^2 + \left(K^* + \frac{K_a}{1-k^2} bb^T\right)\right] x = 0 \tag{22}$$

and, for $Y_{SH} = \infty$ (short-circuited electrodes):

$$\left[Ms^2 + \left(K^* + K_a bb^T\right)\right] x = 0. \tag{23}$$

In the special case of a resistive shunting,

$$\frac{Y_{SH}}{sC} = \frac{1}{sRC}. \tag{24}$$

5. Modal Damping

We now consider the closed-loop characteristic equation in modal coordinates and derive analytical results for the modal damping, for all three cases considered in the previous sections, namely IFF with voltage control, IFF with charge (current) control and resistive shunting. In all cases, the results take the form of a root locus with striking similarities.

IFF, Voltage Control

The development follows closely that of [2]; transforming in modal coordinates according to $x = \Phi z$, assuming normal modes normalized according to $\Phi^T M \Phi = I$; the mode shapes are solution of the eigenvalue problem Eq. (11). Denoting

$$\Phi^T (K^* + K_a bb^T)\Phi = \omega^2 = \mathrm{diag}(\omega_i^2), \tag{25}$$

where ω_i are the natural frequencies of the truss with short-circuited electrodes, Eq. (10) is rewritten

$$\left[Is^2 + \omega^2 - \frac{g}{s+g}\Phi^T (K_a bb^T)\Phi\right] z = 0. \tag{26}$$

The matrix $\Phi^T (K_a bb^T)\Phi$ is in general fully populated; assuming that it is diagonally dominant, and neglecting the off-diagonal terms, it can be rewritten

$$\Phi^T (K_a bb^T)\Phi \simeq \mathrm{diag}(\nu_i \omega_i^2) \tag{27}$$

where

$$\nu_i = \frac{\phi_i^T (K_a bb^T)\phi_i}{\phi_i^T (K^* + K_a bb^T)\phi_i}, \tag{28}$$

is the fraction of modal strain energy in the active strut when the truss vibrates according to mode i. According to the assumption Eq. (27), the open-loop frequency response function (FRF) between the unconstrained expansion δ and the output force y of the collocated sensor can be written ([2], p.61)

$$G(\omega) = \frac{y}{\delta} = K_a \left[\sum_{i=1}^{n} \frac{\nu_i}{1 - \omega^2/\omega_i^2} - 1 \right], \tag{29}$$

where the sum extends to all the modes. Thus, the fractions of modal strain energy ν_i constitute the residues of the modal expansion of the open-loop FRF (Fig. 4). The fact that they are all positive guarantees alternating poles and zeros, as one would expect from a collocated (and dual) actuator/sensor configuration.

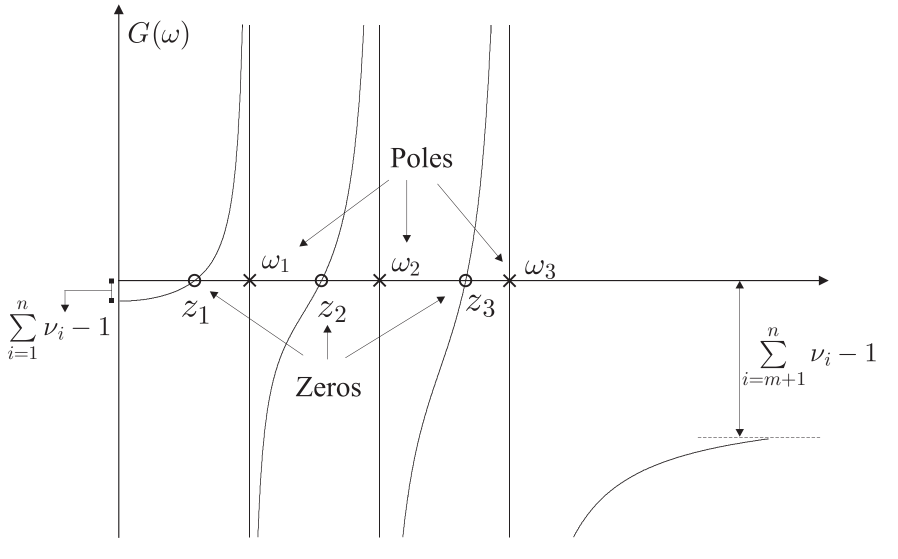

Figure 4. Open loop FRF $G(\omega)$ of the active truss.

It follows also from the assumption Eq. (27) that the eigenvalue problem Eq. (26) reduces to a set of uncoupled equations

$$s^2 + \omega_i^2 - \frac{g}{s+g}\nu_i\omega_i^2 = 0. \tag{30}$$

Denoting

$$z_i^2 = \omega_i^2(1 - \nu_i), \tag{31}$$

Eq. (30) can be transformed into

$$1 + g\frac{s^2 + z_i^2}{s(s^2 + \omega_i^2)} = 0, \tag{32}$$

which shows that every mode follows a root locus with poles at $\pm j\omega_i$ and at $s = 0$, and zeros at $\pm jz_i$ (Fig. 5). Comparing with Eq. (12), the latter are readily identified as the natural frequencies of the structure when the axial contribution of the active strut has been removed. The maximum modal damping is given by

$$\xi_i^{max} = \frac{\omega_i - z_i}{2z_i} \tag{33}$$

and it is achieved for $g = \omega_i\sqrt{\omega_i/z_i}$ [2].

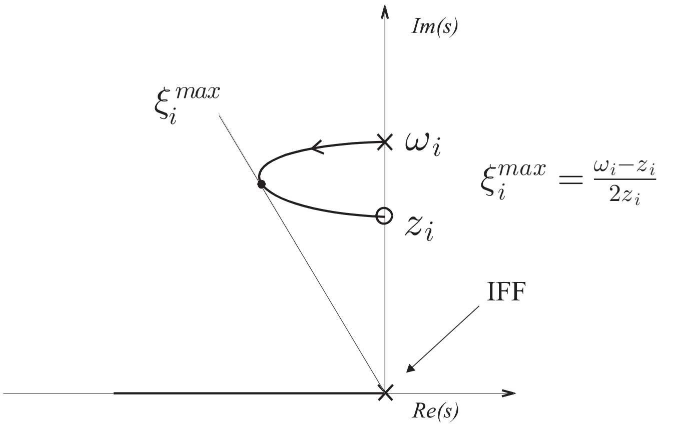

Figure 5. Root locus of the IFF, voltage control (only half of the locus is shown).

IFF, Charge Control

If we consider the charge implementation of the IFF (with a current amplifier), the closed-loop characteristic equation is given by Eq. (17). Transforming in modal coordinates as in the previous section, but using the normal modes of the truss with open electrodes, solutions of the eigenvalue problem (Eq. (18)), the natural frequencies and the fraction of modal strain energy are defined respectively by

$$\Phi^T(K^* + \frac{K_a}{1-k^2}bb^T)\Phi = \Omega^2 = \text{diag}(\Omega_i^2), \tag{34}$$

$$\nu_i = \frac{\phi_i^T(\frac{K_a}{1-k^2}bb^T)\phi_i}{\phi_i^T(K^* + \frac{K_a}{1-k^2}bb^T)\phi_i}. \tag{35}$$

Following the same development as in the previous section, one finds that the closed-loop poles follow the root locus

$$1 + g\frac{s^2 + z_i^2}{s(s^2 + \Omega_i^2)} = 0, \tag{36}$$

which is similar to that of Fig. 5, except that the poles at $\pm j\Omega_i$ correspond in this case to the natural frequencies of the truss with open electrodes; the zeros are identical to those of the previous case.

Resistive Shunting

For resistive shunting, the characteristic equation is given by Eq. (21) with $Y_{SH} = 1/R$; denoting $RC = \varrho$ and transforming in modal coordinates as in the previous sections, one finds that every mode follow the characteristic equation

$$s^2 + \omega_i^2 + \frac{k^2 \nu_i \omega_i^2}{1 - k^2 + 1/\varrho s} = 0 \tag{37}$$

which can be rewritten in a root locus form

$$1 + \frac{s^2 + \omega_i^2}{\varrho(1-k^2)s[s^2 + \omega_i^2 + \frac{k^2}{1-k^2}\nu_i\omega_i^2]} = 0 \tag{38}$$

where $1/\varrho(1-k^2)$ plays the role of the gain in a classical root locus. At the denominator, one recognizes that the poles are located at the natural frequencies of the truss with open electrodes

$$\Omega_i^2 \simeq \omega_i^2 \left(1 + \frac{k^2}{1-k^2}\nu_i\right), \tag{39}$$

while the zeros are at $\pm j\omega_i$, the natural frequencies of the truss with short-circuited electrodes. Figure 6 and Table 1 summarize the results of the three control configurations. Column 4 of Table 1 gives an analytical expression for the maximum achievable modal damping based on the poles and zeros; the approximation given in column 5 is based on

$$\xi_i^{max} = \frac{\omega_i - z_i}{2z_i} \simeq \frac{\omega_i^2 - z_i^2}{4z_i^2}. \tag{40}$$

The influence of the fraction of modal strain energy ν_i and the electromechanical coupling factor k appear much more clearly in the approximate results. Figure 7 shows a chart of the maximum achievable modal damping for the three control strategies, as a function of ν_i and k (based on the exact solution); the contour lines correspond to constant modal damping. Note that:

(i) For the IFF with voltage control, the maximum damping is independent of the electromechanical coupling factor.

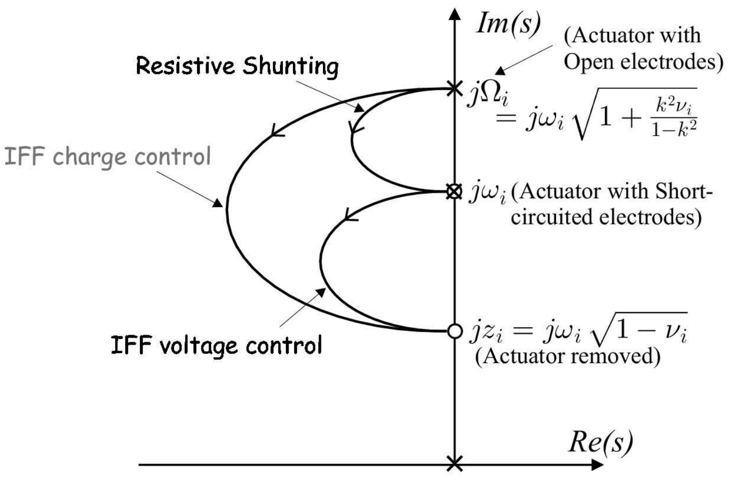

Figure 6. Root locus plots corresponding to various control configurations.

(ii) The IFF with charge control gives always better performances than with voltage control; the advantage increases with k.

(iii) Significant modal damping with resistive shunting can only be achieved when the electromechanical coupling factor is large (say $k \geq 0.7$); the availability of materials with such properties was actually part of the motivations for this study.

6. Inductive Shunting

Inductive shunting was first proposed in [4]; if the piezoelectric transducer is shunted on a RL circuit such that the natural frequency of the electrical circuit is tuned on the natural frequency of one mode, the system behaves like a tuned mass damper [6]. The extension to multiple modes has been addressed in [7], where the use of a set of parallel shunts is suggested; other methods are reviewed in [8]. This section is by no means comprehensive but, once again, the closed-loop poles for inductive shunting are presented in the form of a root locus where the major structural (ν_i), material (k), and electrical (ω_e/ω_i) parameters appear explicitly.

The characteristic equation for an arbitrary passive shunting is given by Eq. (21). Upon transforming in modal coordinates, with the same assumptions as in the previous section, one finds that the characteristic equation for mode i reads:

Table 1. Open-loop poles and zeros and maximum achievable modal damping

Type of Control	Open-loop poles	Open-loop zeros	ξ_i (exact solution)	ξ_i (approximate)
IFF (voltage control)	$\pm j\omega_i$	$\pm jz_i$ $\simeq \pm j\omega_i\sqrt{1-\nu_i}$	$\frac{\sqrt{1-\nu_i}-(1-\nu_i)}{2(1-\nu_i)}$	$\frac{\nu_i}{4(1-\nu_i)}$
IFF (charge control)	$\pm j\Omega_i$ $\pm j\omega_i\sqrt{1+\frac{k^2\nu_i}{1-k^2}}$	$\pm jz_i$	$\frac{\sqrt{1+\frac{k^2\nu_i}{1-k^2}}-\sqrt{1-\nu_i}}{2\sqrt{1-\nu_i}}$	$\frac{\nu_i}{4(1-\nu_i)(1-k^2)}$
Resistive shunting	$\pm j\Omega_i$	$\pm j\omega_i$	$\frac{\sqrt{1+\frac{k^2\nu_i}{1-k^2}}-1}{2}$	$\frac{k^2\nu_i}{4(1-k^2)}$

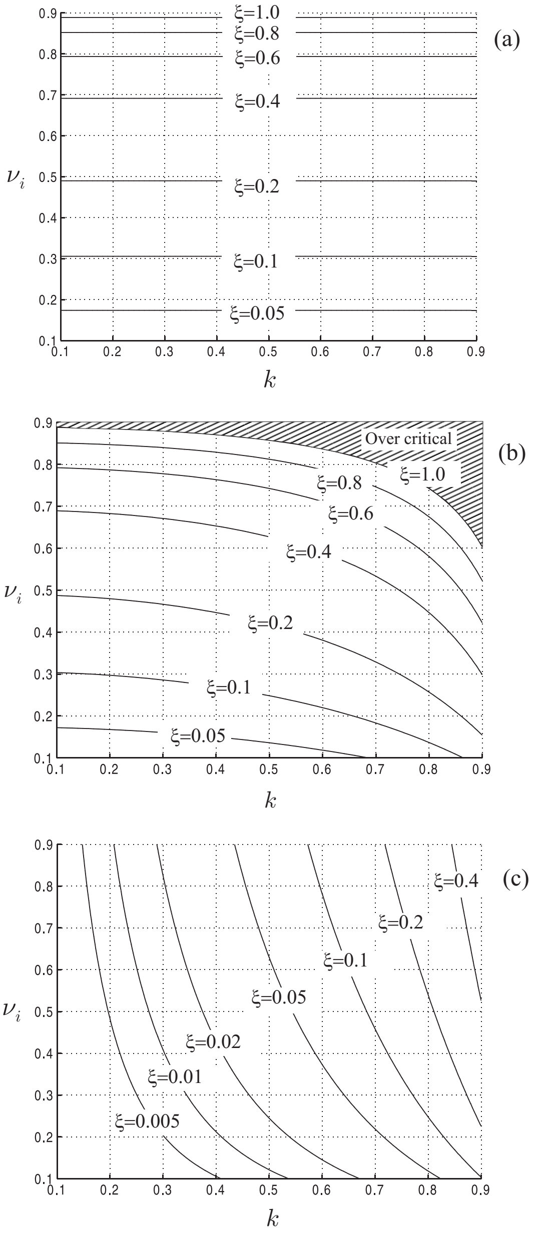

Figure 7. Maximum achievable modal damping as a function of ν_i and k. (a) IFF voltage control, (b) IFF charge control, (c) Resistive shunting.

$$s^2 + \omega_i^2 + \frac{k^2 \nu_i \omega_i^2}{1 - k^2 + Y_{SH}/sC} = 0. \tag{41}$$

If the passive shunt consists of a RL circuit, $Y_{SH} = (Ls + R)^{-1}$ and

$$\frac{Y_{SH}}{sC} = \frac{1}{LCs^2 + RCs}. \tag{42}$$

Upon introducing the electrical natural frequency $\omega_e^2 = (LC)^{-1}$ and the fraction of critical damping ξ_e such that $2\xi_e \omega_e = R/L$, one gets

$$\frac{Y_{SH}}{sC} = \frac{1}{s^2/\omega_e^2 + 2\xi_e s/\omega_e} \tag{43}$$

and the characteristic Eq. (41) becomes

$$s^2 + \omega_i^2 + \frac{k^2 \nu_i \omega_i^2 (s^2 + 2\xi_e \omega_e s)}{(1 - k^2)(s^2 + 2\xi_e \omega_e s) + \omega_e^2} = 0. \tag{44}$$

This equation can, once again, be rearranged in the form of a root locus

$$1 + 2\xi_e \omega_e \cdot \frac{s(s^2 + \Omega_i^2)}{(s^2 + p_1^2)(s^2 + p_2^2)} = 0, \tag{45}$$

in which $2\xi_e \omega_e$ plays the role of the gain. Ω_i is the natural frequency with open electrodes ($\pm j\Omega_i$ are indeed the asymptotic poles when the resistor R becomes very large), and p_1^2 and p_2^2 are solutions of the characteristic equation

$$\frac{s^4}{\omega_i^4} + \frac{s^2}{\omega_i^2}\left[1 + \frac{k^2 \nu_i}{1 - k^2} + \frac{\omega_e^2}{\omega_i^2}\frac{1}{1 - k^2}\right] + \frac{\omega_e^2}{\omega_i^2}\frac{1}{1 - k^2} = 0 \tag{46}$$

which, in addition to ν_i and k, also depends on the tuning ratio ω_e/ω_i between the electrical circuit and the mechanical vibration. The upper half of the root locus consists of two loops whose size depends on the values of the parameters; the depth of the smaller loop in the left half plane tends to be bigger when the tuning ratio is close to 1.

7. Conclusions

Various active and passive ways of damping a piezoelectric truss have been examined; the results have been presented in the common form of a root locus, and analytical formulae have been established for the maximum achievable modal damping. The influence of the modal fraction of strain energy ν_i in the active strut (structural parameter) and the electromechanical coupling coefficient k (material parameter) on the performance has been pointed out.

Acknowledgments

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