A BRIDGE BETWEEN THE MICRO- AND MESOMECHANICS OF LAMINATES: FANTASY OR REALITY?

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Abstract The main topics discussed here are how one can bridge the micro- and mesomechanics of laminates and how this affects the understanding and prediction of localization and final fracture.

Keywords: Damage, laminate, micromechanics, computation

1. Motivations: the Scientific and Industrial Challenges

The last quarter-century has witnessed considerable research efforts in the mechanics of composites in order to understand their behavior and to model or calculate them – the ultimate goal being the design of the materials/structures/manufacturing processes. Even in the case of stratified composites (which are the most studied and, therefore, the best understood), the prediction of the evolution of damage up to and including final fracture remains a major challenge in the modern mechanics of composite materials and structures. Today, the use of stratified composites in the aerospace industry always involves characterization procedures consisting of huge numbers of tests, which shows the low level of confidence in models. A significant improvement in this situation, i.e. a drastic reduction in the number of industrial tests, could be achieved if one could create a real synergy among the approaches on different scales which, today in the case of stratified composites, are followed quite independently of one another. One could jokingly say that there is, on the one hand, the micromechanics of laminates in which one counts cracks and, on the other, the meso- or macromechanics of laminates in which one measures stiffnesses – with only few links between the two. How one can bridge the micro- and mesomechanics aspects and how this affects the understanding and prediction of localization and final fracture are the two main questions discussed here.

2. The Damage Micromechanics of Laminates

What Working Scale?

Up to now, there have been numerous theoretical and experimental works on the micromechanics of laminates (see the two review papers – Nairn and Hu, 1994; Berthelot, 2003, the references at the end of the present paper and, in particular, the book by Herakovich, 1998). In micromechanics, the working scale lies between the dimension of the structure and the diameter of a fiber. In fact, the structure is described as an assembly of cracked interfaces and cracked layers made of a “fiber-matrix” material considered homogeneous or quasi-homogeneous.

Phenomenology on the Microscale

In most practical cases, the sequence of scenarios is as follows.

Scenarios 3 and 4 occur first, leading to rather diffuse damage within the plies and interfaces. Through a percolation phenomenon, transverse

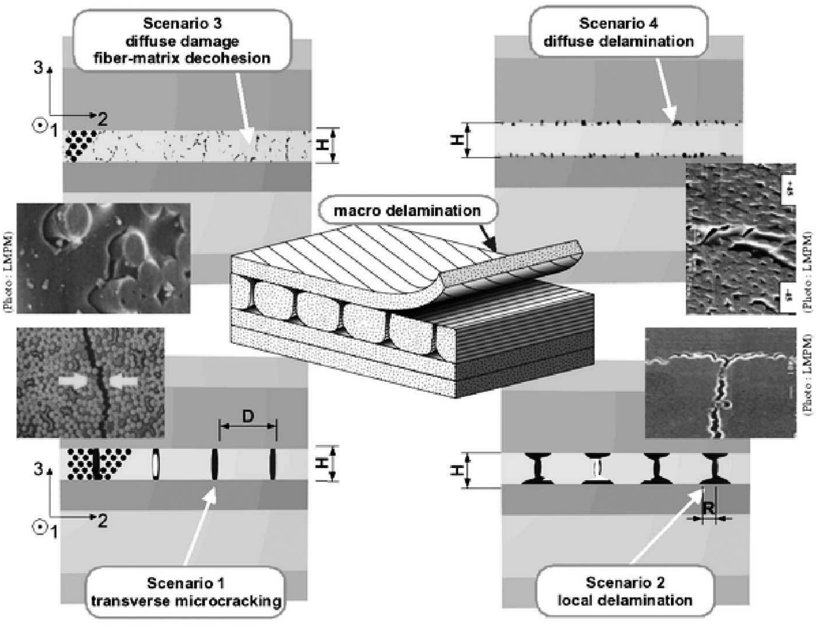

Figure 1. The mechanisms of degradation on the microscale.

microcracking appears; then, Scenario 1 takes over. The competition between transverse microcracking and local delamination ends with the saturation of Scenario 1 and is followed by the catastrophic development of Scenario 2. Ultimately, final fracture occurs with fiber breakage and delamination.

Several Key Points in Micromechanics

Key point 1: the need for Scenarios 3 and 4. Scenarios 3 and 4 are not commonly encountered in micromechanics. Nevertheless, they can be observed by performing the $[45^\circ - 45^\circ]_{2n}$ tension test; a clear and definitive experimental proof was recently given in (Lagattu and Lafarie-Frénot, 2000).

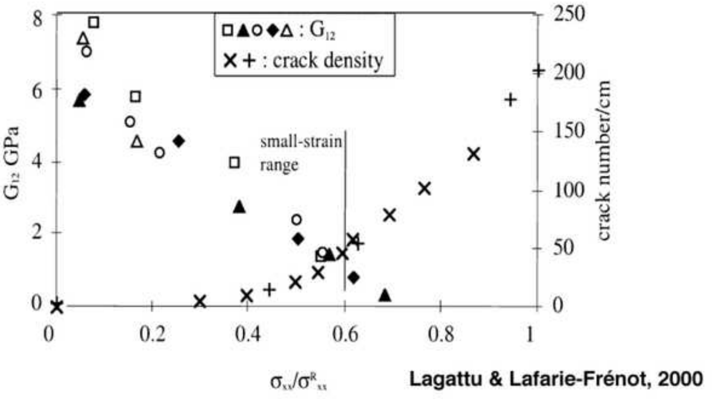

Figure 2. Shear modulus and microcracking density vs. the longitudinal stress for the $[45^\circ - 45^\circ]_{2s}$ tension test.

A major consequence is that there are at least two main damage mechanisms. Therefore, mesomodels with one mesodamage variable per layer are valid only for particular loading types; this is the case of most approaches other than our mesomodel.

Remarks:

- These scenarios are also responsible for the viscoplastic behavior which can be observed on the mesoscale.

- An open question is how to improve one's understanding of the surprising non-percolation phenomenon which occurs under shear on the fiber's scale.

Key point 2: modeling of initiation/propagation – the thickness effect. Most of the basic papers – Garrett and Bailey, 1977; Parvizi et al., 1978; Wang and Crossman, 1980; Boniface et al., 1996; Yang et al., 2003 – are not recent. For tension tests of stacking sequences built with 0° and 90° plies, two main observations have been made: first, the behavior of thick 90° plies is different from that of thin 90° plies; in thick plies, the transverse microcracks always run throughout the width of the specimen, whereas in thin plies they could stop near the edges. The second observation is related to thickness effects (see Fig. 3); the transition thickness $\bar{h}$ is about twice the thickness of the elementary ply.

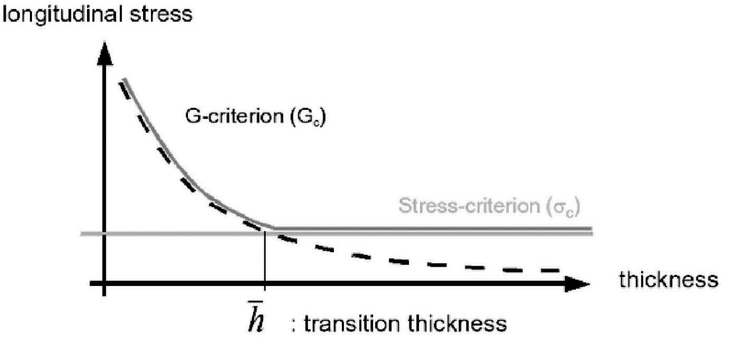

Figure 3. Failure stress vs. the number of 90° plies.

The theoretical explanation is quite old and well-known. Let us consider a flaw in the form of a penny-shaped crack which could propagate either in the longitudinal or in the transverse direction. It has been proven that the transverse energy release rate is much higher than the longitudinal energy release rate and, therefore, that the flaw propagates in the transverse direction, which is the thickness of the layer.

Keypoint 3: microcracking as a stochastic phenomenon. Several probabilistic models (Wang et al., 1984; Fukunaga et al., 1984; Laws and Dvorak, 1988; Manders et al., 1983; Berthelot and Le Corre, 2000) have already been proposed. For high cracking densities, heuristic coefficients have been introduced in order to characterize the non-perfect periodicity (Dvorak and Laws, 1988; Nairn et al., 1993; Ladevèze and Lubineau, 2002). This is necessary in order to have reasonable agreement with experiments (Yalvac et al., 1991).

Here, we support the idea that the process is stochastic, but quasi-independent of the probabilistic law. Let G_c be the critical value associated with the fiber-matrix material. One prescribes a uniform proba-

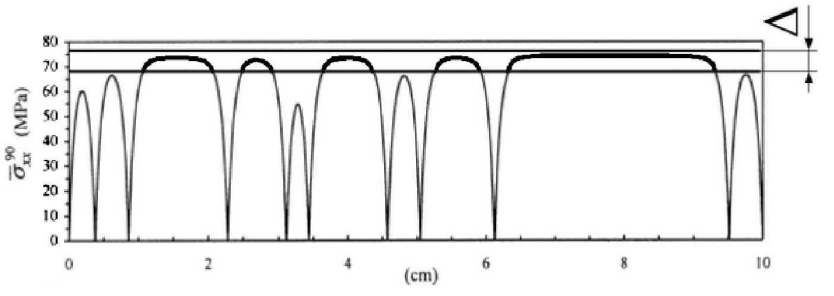

Figure 4. The G-curve as a function of the longitudinal abscissa – domain associated with Δ.

bility density over the domain:

$$\left\{ \underline{M} | \max_{\underline{M}'} G(\underline{M}') - G(\underline{M}) \leqslant \Delta \right\} \tag{1}$$

where G denotes the tunneling energy release rate and Δ a small parameter (see Fig. 4).

Figure 5 shows several samples for different values of Δ; the "mean" curve appears to be insensitive to Δ and to the samples and is a quasi-deterministic curve which is quite different from the curve related to a perfect periodic pattern.

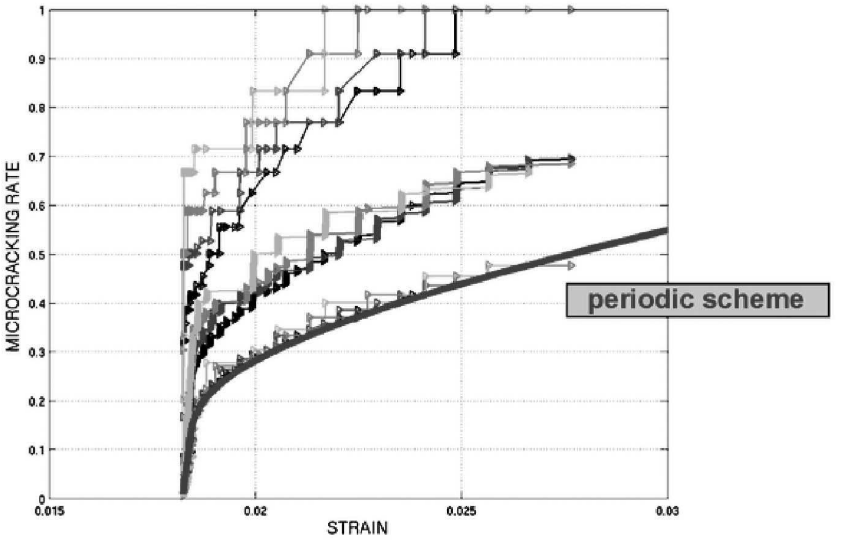

Figure 5. Max., mean and min. values of the microcracking rate vs. the longitudinal strain.

Such a model is in reasonable agreement with experiments (see Fig. 6); the test results for AS4/Hercules 3501-6 are given in Nairn and Hu, 1994.

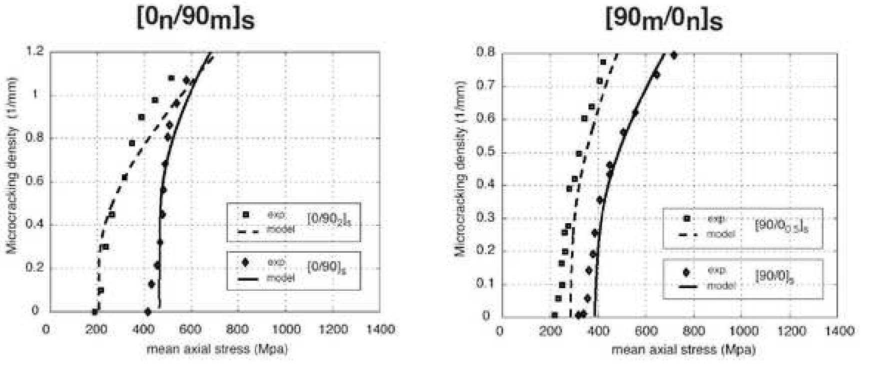

Figure 6. Comparison of tests vs. model.

3. A Bridge Between the Micro- and Mesomechanics

A rather complete bridge was developed in Ladevèze and Lubineau, 2001, 2002; Ladevèze et al., 2004.

The Method for Bridging the Two Mechanics

The real structure described on the microscale is subjected to a given loading. Its analysis constitutes a two-scale problem which we solve using a classical two-step scheme. The first step consists in calculating the large-wavelength part of the solution – the mesosolution – by solving what one calls the homogenized structure. In the second step, the micropart (i.e. the small-wavelength part) of the solution is determined in terms of the mesoquantities. This approach is applied to two basic problems which are representative of all engineering situations: the basic ply problem and the basic interface problem. These problems are described in Figs. 7 and 8. Periodic conditions are applied and elastic behavior is assumed. Classically, the solution of the homogenized problem is built first. For the real structure, this solution needs to be corrected because the residuals associated with discontinuities, which are assumed to be locally uniform, must be equilibrated.

The equivalence should hold true for any set of residuals which can be written in terms of mesoquantities.

The fundamental micro-meso link which defines what one calls the homogenized structure is exactly true for the two basic problems; it can

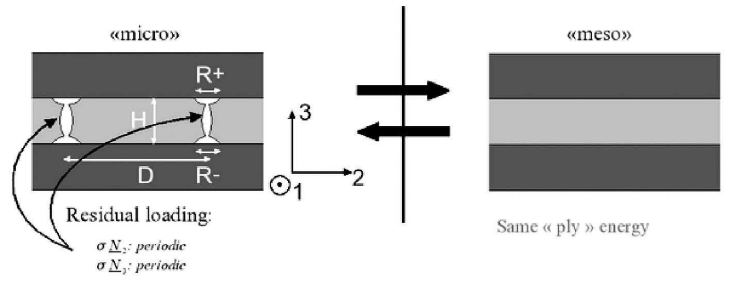

Figure 7. The basic ply problem.

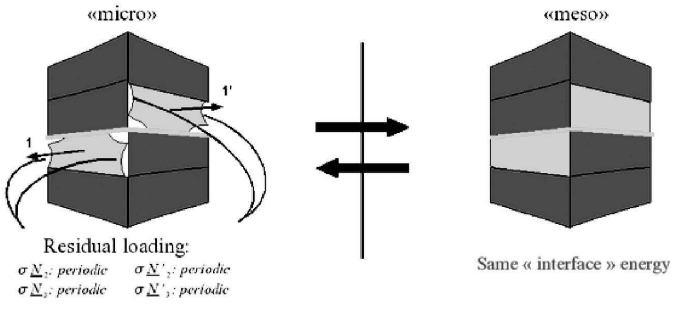

Figure 8. The basic interface problem.

be written as:

$$\begin{aligned} &\forall \Gamma \;\; \boldsymbol{\Pi}\,\boldsymbol{\varepsilon}_{\text{meso}}\boldsymbol{\Pi} = \langle \boldsymbol{\Pi}\,\boldsymbol{\varepsilon}_{\text{micro}}\boldsymbol{\Pi} \rangle, \\ &\boldsymbol{\sigma}_{\text{meso}}\underline{N}_3 = \langle \boldsymbol{\sigma}_{\text{micro}}\underline{N}_3 \rangle, \\ &\left(\langle \bullet \rangle = \frac{1}{\text{mes}\Gamma} \int_\Gamma \bullet \, dS. \right) \end{aligned} \tag{2}$$

Γ is any cross section orthogonal to $\underline{N}_3$ and compatible with the periodicity associated with the layer or the interface containing Γ. In practice, for high crack densities, Γ could be replaced by any cross section which is wide with respect to the ply's thickness. $\boldsymbol{\Pi}$ is the projector onto the plane orthogonal to $\underline{N}_3$.

The resolution of the microproblem has been a major issue in micromechanics; quasi-analytical approximations have been derived (Hashin, 1985; Nairn, 1989; Aboudi et al., 1988; Nuismer and Tan, 1998; Zhang et al., 1992; McCartney, 1992, 2000; Schoeppner and Pagano, 1998; Varna et al., 1992; Berthelot et al., 1996; Selvarathimam and Weitsman, 1999 ...) Here, we follow another calculation method which, in fact, is a functional analysis: first, the solution is determined in terms of material operators which depend on the microdamage variables and the different additional parameters. Then, these operators are calculated for all values of the parameters and microdamage variables (in practice, in the range $\rho \in [0 \ \ 0,7]$ $\tau \in [0 \ \ 0,4]$). Consequently, a very large number of calculations are being performed.

Virtual Testing of the Ply

- The basic ply problem

  The mesoenergy of the cracked ply can be written:

$$e_{\text{meso}}(\llbracket \varepsilon_{\text{meso}} \rrbracket, \varpi_{\text{meso}} \underline{N}_3) = \frac{1}{2} T_r [\mathbf{H} \llbracket \varepsilon_{\text{meso}} \rrbracket \llbracket \varepsilon_{\text{meso}} \rrbracket] + \frac{1}{2} \varpi_{\text{meso}} \underline{N}_3 \cdot \mathbf{A} \varpi_{\text{meso}} \underline{N}_3 + \varpi_{\text{meso}} \underline{N}_3 \cdot \mathbf{B} \llbracket \varepsilon_{\text{meso}} \rrbracket \quad (3)$$

  where $\mathbf{H}$, $\mathbf{A}$ and $\mathbf{B}$ depend on the microdamage variables and on the parameters of the upper and lower parts. Numerically, it has been proved that:

  Fundamental property:

  Operators $\mathbf{H}$, $\mathbf{A}$ and $\mathbf{B}$ are quasi-intrinsic homogenized operators. In practice, they do not depend on the parameters of the upper and lower parts.

  Additional results:

  Mesoenergy has the following remarkable expression:

$$e_{\text{meso}}(\llbracket \varepsilon_{\text{meso}} \rrbracket, \varpi_{\text{meso}} \underline{N}_3) - e^0_{\text{meso}} = -\frac{1}{2}\left[\frac{d_{22}}{1-d_{22}} \frac{\sigma_{22}^2}{E_2^0} + \frac{d_{12}}{1-d_{12}} \frac{\sigma_{12}^2}{G_{12}^0}\right] - \frac{1}{2}\left[\frac{d_{33}}{1-d_{33}} \frac{\sigma_{33}^2}{E_3^0} + \llbracket \varpi \underline{N}_3 \rrbracket \cdot \mathbf{C} \llbracket \varpi \underline{N}_3 \rrbracket\right] \quad (4)$$

  All the stresses involved are mesoquantities; moreover, the mesodamage variables d_{22}, d_{12}, d_{33} and $\mathbf{C}$ (which is diagonal) can be calculated very easily in terms of the microdamage variables:

$$d_{ij}(\rho, \tau^+, \tau^-) = f_{ij}(\rho, \tau^+) + f_{ij}(\rho, \tau^-). \quad (5)$$

- The extended basic ply problem

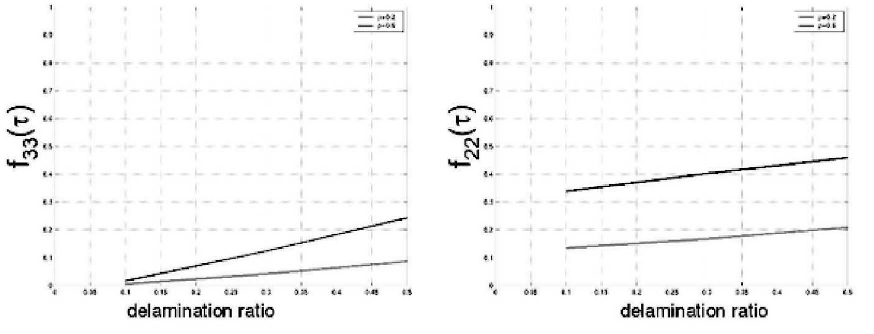

Figure 9. The functions f_{33} and f_{22} related to the basic ply problem.

Now, the cracked ply being studied is located between two adjacent cracked plies. The fundamental property is still valid; in practice, the ply's mesomodel depends only on the ply and its adjacent interfaces. A similar property holds for the damage variables. Consequently, the ply's mesomodel can be derived easily through a sequence of calculations.

Virtual Testing of the Interface

The basic interface problem is a 3D problem which can be approximated with good accuracy by two 2D problems. The mesoenergy can be written:

$$E_{\text{meso}}(\varpi_{\text{meso}} \underline{N}_3) = \frac{1}{2} \varpi_{\text{meso}} \underline{N}_3 \cdot \mathbf{D} \varpi_{\text{meso}} \underline{N}_3 \tag{6}$$

where $\mathbf{D}$ depends on the microdamage variables and on the parameters of the upper and lower parts. Numerically, it has been proved that:

<u>Fundamental property</u>:

In practice, operator $\mathbf{D}$ does not depend on the parameters of the lower and upper parts. The interface's mesomodel depends on the interface and its adjacent plies.

<u>Additional results</u>:

The mesodamage quantity D_{33} related to the opening delamination mode (Mode I) depends only on the total delamination ratio. Figure 10 shows the other mesodamage quantities D_{13} and D_{23} associated with Modes II and III, which quantifies the interaction between intra- and interlaminar mesodamage.

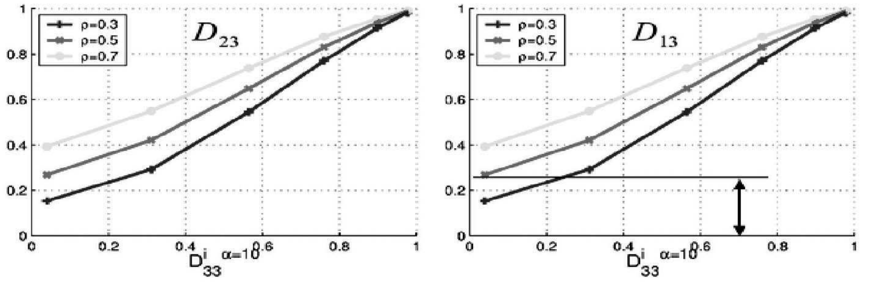

Figure 10. Interface mesodamage quantities D_{13} and D_{23} related to Modes II and III.

4. Perspectives for Damage Analysis

An Enhanced Mesomodel

The standard mesomodel has been extensively used for the resolution of impact and quasi-static engineering problems. We are now developing an enhanced version in order to improve the prediction of delamination. This development is an application of the bridge which has just been presented. The interface's damage mesomodel is made nonlocal by coupling it with the microcracking of the adjacent plies. Another improvement consists in using the micro-meso relations to describe damage in terms of micromechanics.

This mesomodel needs to go through the identification procedure again. A further improvement would be to work with the true, complete mesomodel, which is nonlocal both for the interface and for the ply.

A Computational Damage Micromodel of Laminates

The starting point is an initial state including thermal stresses, which can be calculated by simulation of the curing process or, classically, by applying a negative temperature variation ΔT.

The model is discrete; in fact, it is hybrid. The fiber-matrix material follows the continuum mechanics framework; more precisely, its behavior is given by the mesomodel restricted to (visco)plasticity and diffuse damage (Scenarios 3 and 4). The cracked surfaces related to microcracking and delamination follow what is called "finite fracture mechanics" (Hashin, 1996), for which we introduced minimum cracked surfaces (Fig. 11). Initiation and propagation criteria are considered

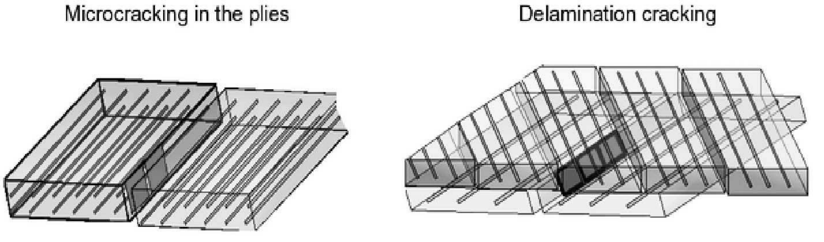

Figure 11. The minimum cracked surfaces.

only for microcracking. For fiber breakage, a minimum cracked volume, whose height is the thickness of the ply, is prescribed.

Remarks:

- When a surface is cracked, unilateral contact conditions with friction are applied.

- Mode I is distinct from Modes II and III.

- Stochastic modeling is used for the critical parameters of the material.

This rather simple computational micromodel could be used as a reference. It is *a priori* in good agreement with experimental micro- and macroinformation. However, computational difficulties pose a serious problem with this model. If we consider a low-velocity impact problem (T300/914 24 plies), the calculation procedure leads to the following numbers of degrees of freedom for the largest problem to be solved:

– classical finite element method: 2.10^{10} DOFs

– multiscale computational strategy: 4.10^{8} DOFs (one macroscale) and 10^{6} DOFs (two macroscales).

Consequently, a research challenge consists in deriving alternative computational strategies capable of solving such engineering problems. This is currently a hot issue, known as “multiscale computational strategies including uncertainties”, for which several proposals have already been made (in particular Devries et al., 1989; Feyel, 2003; Fish et al., 1997; Kouznetsova et al., 2002; Ladevèze et al., 2001; Ladevèze and Nouy, 2003; Zohdi et al., 1996).

Conclusion

Let us come back to the central question discussed in this paper: is there a bridge between the micro- and mesomechanics of laminates? The answer is yes, but the mesomechanics of laminates is not a simple field, because it is nonlocal.

Among the applications of this bridge are multiscale computational approaches for the prediction of final fracture, which require further research, especially in computational mechanics, prior to becoming engineering tools.

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