Continuous-time filters

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Applications and problems

• Applications
  • Anti-aliasing filters
  • Video and HF filters: hard-disk drives
  • Channel select filters
  • Low-power filters

• Problems:
  • Tuning for high precision: mismatch < 5 %
  • Distortion: THD < -60 dB
  • Low power supply voltages
  • High quality factors: Q > 50 ?
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- Active RC filters
  - MOSFET-C filters
  - GmC filters
  - Comparison

      J. Silva-Martinez, Kluwer 1993,
      W. Dehaene, JSSC, July 1997, 977-988
Active RC filters

Opamps and passive components (R, C)

Advantages:
- S/N up to 100 dB
- THD very low < -90 dB

Disadvantages:
- Opamps: only for low frequencies
- Errors on R, C ≈ 20% 
  >> tune C’s

Opamps and passive components (R, C):
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Ref.: Silva-Martinez, Dehaene, ..
MOST resistors

On-resistance:

\[ R_{\text{on}} = \frac{1}{\beta (V_{GS} - V_T)} \]

\[ \beta = KP \frac{W}{L} \]

\[ i_{DS} = KP \frac{W}{L} [(V_{GS} - V_T)v_{DS} - \frac{v_{DS}^2}{2}] \]

Diagram:

- Linear region: \( V_{DS} > V_{GS} - V_T \)
- Saturation region: \( V_{DS} > V_{GS} - V_T \)

HD2

Willy Sansen 10-05 196
Examples of differential MOST-R’s

\[ i - i' = \frac{2v_X}{R} \]

(c)

\[ V'_C + \frac{v_X + v_Y}{R} \]

(d)

\[ i = \frac{v_X - v_Y}{R} \]

partial cancellation of even nonlinearity

\[ i - i' = 2v_X\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \]

(h)

cancellation of even and odd nonlinearities

Ref. Tsividis JSSC Feb.86, 15-30; Ma.94, 166-176
From active RC to MOSFET-C filter

Ref. Tsividis JSSC Feb.86, 15-30
Large $R_{ON}$ values at high frequencies

For low-frequency low-pass filter with $f_{-3dB}$

$$f_{-3dB} = \frac{1}{2\pi R_{on}C} \approx \frac{KP \ W/L \ (V_{GS}-V_{T})}{2\pi C}$$

For $f_{-3dB} = 4$ kHz; $KP = 60 \ \mu A/V^2$; $V_{GS}-V_{T} = 1$ V; $W = 2 \ \mu m$; $C = 10$ pF
$R_{on} = 4 \ M\Omega$. For matching $W = 2 \ \mu m$: $L \approx 500 \ \mu m$ ! The area is $10^{-5}$ cm$^2$

For $C_{ox} = 5.10^{-7}$ F/cm$^2$ (0.35 $\mu m$); $C_{GS} = 5$ pF;
High-frequency limit at $\approx 8$ kHz or $f_T \approx 8$ kHz  !!!!!!
LC ladder filter

Ref. Banu
JSSC Dec.85,
1114-1121
Fifth-order low-pass filter

Ref. Tsividis, JSSC Feb.86, 15-30
Fifth-order elliptic low-pass filter

Ref. Tsividis
JSSC Feb.86, 15-30
PLL tuning

Problems:
Master/slave for each pole/zero
VCO + PLL at $f_c$
Signal feedthrough at $f_c$

Ref.
Banu JSSC Dec.85, 1114-1121
Krummenacher, JS SC June 88, 750-758
Khoury, JSSC Dec.91, 1988-1997
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Ref.: Silva-Martinez, Dehaene, ..
Some GmC filters

Single-ended GmC filters

Fully-differential ...
GmC filter definition

**Opamp**
Operational amplifier

\[ A_v = \frac{v_{OUT}}{v_{IN}} \]

**OTA**
Operational Transconduct. amplifier

\[ A_g = \frac{i_{OUT}}{v_{IN}} \]

\[ A_v = A_g \cdot R_L \]

Adv.: High freq. operation
Easy tuning

Disadv.: Distortion
Mismatch errors
Parasitic C’s (low Q)
Simple GmC filters

\[ \frac{v_{\text{OUT}}}{v_{\text{IN1}} - v_{\text{IN2}}} = \frac{g_m}{sC} \]

\[ \frac{v_{\text{OUT}}}{v_{\text{IN}}} = \frac{g_mR}{1 + sRC} \]
Simple GmC filters

\[
\frac{V_{OUT}}{V_{IN}} = \frac{g_{m1}}{g_{m2} + sC}
\]
Simple fully-differential GmC filters

Integrator

\[ \text{G}_m, C_L \]

Resistor

\[ \text{C}_p \]

Lossy integrator

Sensitive to parasitic capacitances

Biquadratic cell

\[ \text{G}_m, C_1, C_2, \text{G}_m, \text{G}_m \]
Voltage-mode & current-mode filters
A differential pair as a transconductor

\[ \text{IM}_3 = 3\text{HD}_3 = \frac{3}{32} U^2 \]

\[ U = \frac{V_{id}}{V_{GS} - V_T} \]

\[ \text{Max.} \ V_{idptp} \approx 2 \sqrt{2} (V_{GS} - V_T) \]

\[ \text{IP}_3 \approx 3.3 (V_{GS} - V_T) \]

\[ \text{HD}_3 = -60 \text{ dB for} \ V_{id} = 1 \text{ V requires} \ V_{GS} - V_T = 6 \text{ V} \]
Amplifier or transconductor?

Transconductor:
Wide input range: low distortion
Small gain $g_m$
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Increasing the IP$_3$ by feedback

\[ IP_3 \approx 3.3 \left( V_{GS} - V_T \right) \left( 1 + g_{m1} R \right)^2 \]

\[ HD_3/n^2 \quad n = 1 + g_{m1} R \]

HD$_3$ = -60 dB for $V_{id} = 1$ V requires $V_{GS} - V_T = 0.38$ V and $g_{m1} R = 3$ !!!
Increasing the IP₃ by FB and high loop gain

Additional local FB

More FB with opamps

Willy Sansen 10-05 1925
Tuneable resistances
By tuneable feedback

\[ IP_3 \approx 3.3 (V_{GS} - V_T)n^2 \]

\[ \frac{HD_3}{n^2} \]

\[ n = 1 + \frac{g_{m1}}{g_{m2}} \]

Ref. Torrance et al. CAS Nov.85, 1097-1104
By nonlinear feedback (input)

$$g_{m\text{tot}} = \frac{I_{\text{bias}}}{n (V_{GS1} - V_T)}$$

$$n = 1 + \frac{\beta_1}{4\beta_2} \frac{\text{HD}_3}{n^2}$$

$$I_{P3} \approx 3.3 \left( V_{GS1} - V_T \right) n^2$$

No extra current
No extra CM node!
But limited $V_{GS}$-$V_T$!

Ref. Krummenacher JSSC June 88, 750-758
By nonlinear feedback (as load)

\[ \frac{2R_{ind}}{V_{outd}} \approx \frac{2n}{g_{m1}} \]

\[ A_v = \frac{n g_{m3}}{g_{m1}} \]

\[ n = 1 + \frac{\beta_3}{4\beta_2} \]

Ref. Menolfi JSSC July 97, 968-976
Low-distortion combination: power!

\[ V_{id}/2 \]

\[ I_{bias} \]

\[ M1 \]

\[ M2 \]

\[ V_{id}/2 \]

\[ -V_{id}/2 \]

\[ I_{out} \]

\[ -I_{out} \]
Reduced distortion by cross-coupling

Ref. Silva-Martinez JSSC July 91, 946-955
Comparison (simulations)

\[ G_m = 150 \mu A/V \quad V_{dd}=\pm 2.5V \]

**THD (%)**

- **Torrance N=3 (I=900\mu A)**
- **Krummenacher (I=300\mu A)**
- **Presented (I=480\mu A)**

**Input Voltage (V_{ptp})**

0.0 0.5 1.0 1.5 2.0 2.5
Measured THD for transconductor

THD ≈ -60 dB
2.4 V_{pp}
Input Voltage

Ref. Silva-Martinez JSSC July 91,946-955
Linear transconductor with opamps

Ref. Chang JSSC March 97, 388-397
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Parallel differential pairs with offset Voltages

\[ \frac{v_{id}}{2} - \frac{v_{id}}{2} \]

\[ I_B \]

\[ V_G \approx 1.3 \ kT/q \]
\[ \approx 34 \ mV \]

Ref. Gilbert, JSSC
Dec. 82, 1179-1191
Voormann ECCTD 83
Tanimoto, .. JSSC
July 91, 937-945
Transfer characteristics of parallel diff. pairs

\[
i_{\text{CE}} = \frac{I_B}{1 + \exp(-qV_{\text{Id}}/kT)}
\]

\[V_{GG} = 34 \text{ mV}\]
Parallel diff. pairs with different transistor sizes

\[ \text{Bidirectional operation:} \quad i_{\text{out}} = -i_{\text{out}} \]

\[ \text{Ref. in bipolar:} \]
- Gilbert, JSSC, Dec. 82, 1179-1191
- Voormann, ECCTD 83 ESSCIRC 85
- JSSC, Aug. 00, 1097-1108
- Tanimoto, JSSC July 91, 937-945
- De Veirman, JSSC March 92, 324-331

Bipolar: \( n \approx 4 \)
Paralleling four differential pairs

5th order filter

Input range 160 mV_{ptp} (1\% THD)

Ref. Tanimoto,.. JSSC July 91, 937-945
Dual-input transconductor

7th order filter
2 … 10 MHz

Input range
96 mV_{ptp} (1\% THD)

Ref. De Veirman, JSSC, March 92, 324-331
Parallel diff. pairs with different transistor sizes

\[ \frac{i_{\text{out}}}{v_{\text{id}/2}} = \frac{n}{1} \]

Parameters:
\[ \alpha = \frac{I_{B2}}{I_{B1}} \]
\[ v = \frac{V_{\text{GST}1}}{V_{\text{GST}2}} \]
\[ \frac{V_{\text{GST}}}{V_{GS}-V_{T}} \]

\[ n = \alpha v^2 \]

MOST: \( n \approx 5 \)

Ref. in CMOS:
Nedungadi, CAS
Oct 84, 891-894
Voorman, JSSC
Aug. 00, 1097-1108
Luh, ESSCIRC 00
Cross-coupling for linearity and swing

Ref. Luh, USC, ESSCIRC 2000, 72-75
Multiplier or Amp. with distortion cancellation

Parameters:
\[ \alpha = \frac{I_{B2}}{I_{B1}} \]
\[ v = \frac{V_{GST1}}{V_{GST2}} \]
\[ V_{GST} = V_{GS} - V_T \]

Ref. Gilbert, JSSC Dec. 68, 365-373
Cross-coupling and source resistors

Ref. Prodanov, ESSCIRC 2001, 488-491

\[ \frac{v_{id}}{2} \rightarrow M1 \rightarrow \text{i}_{\text{out}} \rightarrow M3 \rightarrow \text{-i}_{\text{out}} \rightarrow \frac{-v_{id}}{2} \]

\[ I_{\text{bias}} \]
Cross-coupling and source followers

\[ i_{\text{out}} = \frac{v_{id}}{2} \]

\[ -i_{\text{out}} = \frac{-v_{id}}{2} \]

Ref. Van Engelen, JSSC Dec.99, 1753-1764
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Linearity CMOS amplifier

\[ V_{in} = V_{out} = \frac{V_{DD}}{2} \]

if \( K'_n \frac{W_n}{L_n} = K'_p \frac{W_p}{L_p} \)

very linear!
Linearized transconductors

Ref. Voorman, JSSC, Aug.2000, 1097-1108
Transconductor for High Frequencies (2 nodes)

I_{od} = I_{o1} - I_{o2} = g_{md} V_{id}

\[ R_{DM1} = \frac{1}{g_{m5}-g_{m6}} \]

\[ R_{DM2} = \frac{1}{g_{m4}-g_{m3}} \]

\[ R_{CM1} = \frac{1}{g_{m5}+g_{m6}} \]

\[ R_{CM2} = \frac{1}{g_{m4}+g_{m3}} \]

Ref. Nauta JSSC Febr.92,142-146
Transconductors with linear MOSTs

\[ V_{DS1} = R_D I_D \approx 0.2 \, V \]

\[ I_{DS1} = \beta_1 V_{DS1} (V_{GS1} - V_T) \]

\[ g_{m1} = \beta_1 V_{DS1} \text{ is constant over wide range!} \]

Alini, JSSC, Dec.92, pp.1905-1915
Alternative solutions

Larger tuning range
Controls $V_{DS}$
$V_{DS\text{min}} \approx 0$
$V_{\text{tuning}}$ down to 0

Smaller tuning range
Controls $V_{GS} - V_T$
$V_{GST\text{min}}$ limited by linearity
$V_{\text{tuning}}$ from $V_T$ up
Pseudodifferential trans. with linear MOSTs

Biasing imposed by previous circuit!

No rejection of CM signals (CMRR = 0 dB)

Ref. Alini, JSSC, Dec.92, pp.1905-1915
Transconductors with linear MOSTs

\[ g_{m1} = \beta_1 V_{DS1} \]

is constant over wide range!

Ref. Laber, JSSC, April 93, 462-470
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Ref.: Silva-Martinez, Dehaene, ..
Gm-R-C versus Gm-C filters
Gm-R-C filters

\[ f_0 \approx \frac{1}{2\pi} \sqrt{\frac{g_{m1}g_{m2}}{C_1C_2}} \]

if \( f_0 \ll f_{par} \)

\[ Q \approx \frac{g_{m2}}{g_2 + g_{o2}} \]

if \( g_1 \approx 0 \) (cascodes)

Ref. Silva-Martinez JSSC July 91,946-955
Tunable R to tune Q

\[ R_{OUT} \approx \frac{1}{KP_1 \frac{W_1}{L_1} (V_C - V_{GS2})} \]
Tuning systems: transconductance tuning

\[ I_{\text{ref}} = V_{\text{ref}} / R \]
\[ I_{\text{ref}} = g_m V_{\text{ref}} \]
\[ g_m = 1 / R \]

\[ V_{\text{OUT}} = v_{\text{tune}} \]
Tuning systems: frequency tuning

\[ R = \frac{1}{C_{f_c}} \]

\[ I_{\text{ref}} = C_{f_c} V_{\text{ref}} \]
\[ I_{\text{ref}} = g_m V_{\text{ref}} \]
\[ g_m = C_{f_c} \]

Ref. Viswanathan, JSSC Aug. 82, 775-778
Silva, JSSC Dec. 92, 1843-1853
Fully-differential tuning system realization

Ref. Chang, JSSC March 1997, 388-397

Willy Sansen 10-05 1960
Tuning systems: frequency tuning with low $f_c$

Charge balancing: No crosstalk

$$Q_{gm} = C \frac{I_B}{g_m}$$

$$Q_{IBN} = \frac{I_B}{N} T_c$$

$$\frac{g_m}{C} = N f_c$$

Ref. Silva
Fully-differential frequency tuning system

\[
g_m \frac{I_B}{C} = Nf_c
\]

Ref. Silva, JSSC Dec. 92, 1843-1853
Tuning systems : Q tuning

Unity-pulse response Biquad :

$$H(t) = \frac{1}{\sqrt{1 - \frac{1}{4Q^2}}} \exp\left(-\frac{t \cdot BW}{2}\right) \sin\left(\sqrt{1 - \frac{1}{4Q^2}} \omega_o t + \theta\right)$$

No HF PLL or VCO! Detection at $f_c$!
Q tuning with active resistor

\[ R_{\text{tune}} \]

\[ C \]

\[ 2C \]

\[ \text{OTA1} \]

\[ \text{OTA2} \]

\[ \text{envelope detector} \]

\[ \text{AMP} \]

\[ \text{LPF} \]

\[ VQ \]
Comparison of 10.7 MHz filters

<table>
<thead>
<tr>
<th></th>
<th>SC</th>
<th>OTA-C</th>
<th>Gm-RC</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_c$ (BW = 250 kHz)</td>
<td>10.7 MHz</td>
<td>12.5 MHz</td>
<td>10.7 MHz</td>
</tr>
<tr>
<td>Order filter</td>
<td>6</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>$V_{in} @ IM3= 1%$</td>
<td>$0.24 \text{ V}_{\text{RMS}}$</td>
<td>$0.32 \text{ V}_{\text{RMS}}$</td>
<td>$0.71 \text{ V}_{\text{RMS}}$</td>
</tr>
<tr>
<td>DR @ IM3= 1%</td>
<td>34 dB</td>
<td>51 dB</td>
<td>68 dB</td>
</tr>
<tr>
<td>Power (± V)</td>
<td>500 mW(± 5)</td>
<td>360 mW(± 6)</td>
<td>220 mW(± 2.5)</td>
</tr>
<tr>
<td>Chip area</td>
<td>2 mm²</td>
<td>7.8 mm²</td>
<td>6 mm²</td>
</tr>
</tbody>
</table>
Biquad for 7th-order Filter at 50 MHz

\[ C = \sum C_{\text{parasitic}} \]

\[ g_{m2}^* = g_{m2} + \sum g_o \]

\[ A_v = \frac{g_{m3}}{g_{m1} (\gamma^2+1)} \]

\[ Q = \frac{\sqrt{(\gamma^2+1)}}{2\gamma} \]

\[ f_o = \frac{1}{2\pi \tau} \frac{\sqrt{(\gamma^2+1)}}{\gamma} \]

\[ \tau = \frac{C}{g_{m2}^*} \gamma = \frac{g_{m2}^*}{g_{m1}} \]

Biquad with matched nodes

Ref. Dehaene JSSC July 97, 977-988

Willy Sansen 10-05 1966
Tuning system for Q: conductance ratio $\gamma$

$\Phi_1$ closed: $\gamma$ tuning mode

$\gamma = \frac{g_{m2}^*}{g_{m1}}$

$V_{n+,n^-} = \frac{g_{m1}}{g_{m2}^*} g_{OTA} V_{\text{ref}} = kV_{\text{ref}} g_{OTA}$

$\Phi_1$ closed: offset calibration mode for all OTA’s

$V_{\text{ref}} = kV_{\text{ref}} g_{OTA}$

$\Rightarrow \gamma = \frac{1}{k}$

Ref. Dehaene JSSC July 97, 977-988

Willy Sansen 10-05 1967
Tuning system for the ratio of time constants

Charge balancing:

\[ V_{1,\text{int}} = V_{2,\text{int}} \quad \text{or} \quad \frac{V_{\text{ref}}}{\gamma_1} \frac{g_m}{C_{\text{int}}} \tau_1 = \frac{V_{\text{ref}}}{\gamma_2} \frac{g_m}{k_{12} C_{\text{int}}} \tau_2 \quad \Rightarrow \quad \frac{\tau_2}{\tau_1} = k_{12} \frac{\gamma_2}{\gamma_1} \]
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J. Silva-Martinez, Kluwer 1993,
W. Dehaene, JSSC, July 1997, 977-988
Signal to Noise + Distortion ratio

- RC (GBW)
- MOST-C (GBW)
- SC (settling)
- \( g_mC \) (linearity)
- SI (mismatch)
- \( g_mC \)

Frequency (f) ranges from 10 kHz to 1 GHz.

- + Disto - Tuning
- + Tuning - Disto
- + Tuning - HiFr
- + Power, Disto
- + Tuning - Disto
- + HiFr
- +HiFr - DR
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