

## Appendix 5.1: Improving the CRT Deflection Field

In the vacuum of a cathode ray tube (CRT) a homogeneous electric field within the ‘electron gun’ of strength  $E = V_a/d_{ka}$  exerts a force  $F = q_e E$  on an electron emitted by the hot cathode, causing it to accelerate at the rate:

$$a = \frac{F}{m_e} = \frac{q_e E}{m_e} = \frac{q_e V_a}{m_e d_{ka}} \quad (\text{A5.1.1})$$

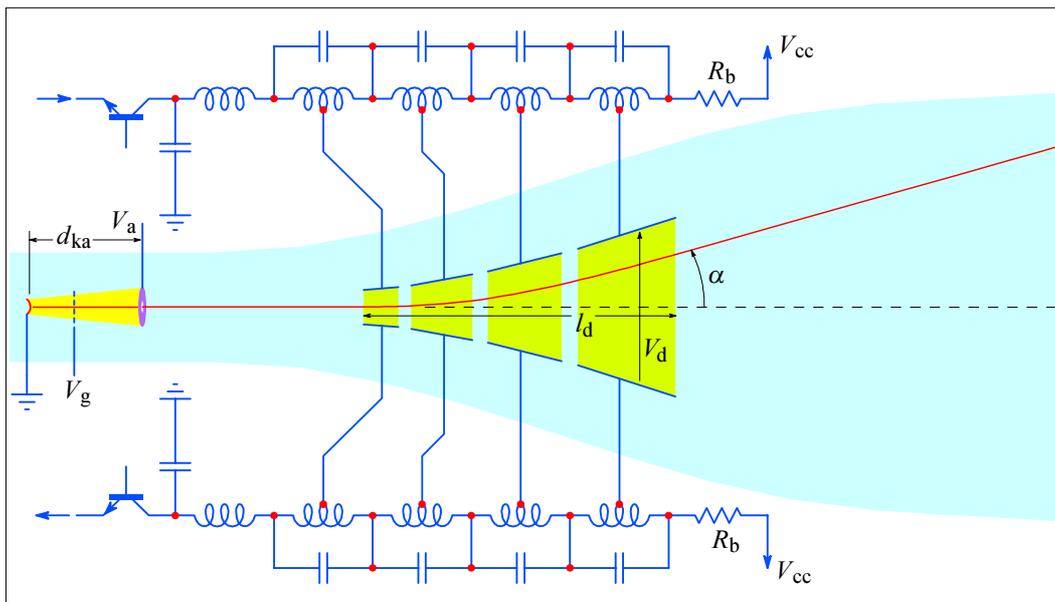
where:  $q_e = -1.602 \times 10^{-19}$  [As] is the electron’s charge;

$m_e = 9.1095 \times 10^{-31}$  [kg] is the electron’s (rest) mass;

$V_a$  [V] is the voltage potential between the anode and cathode;

$d_{ka}$  [m] is the field length (the distance from the cathode to the anode).

We have neglected the intensity modulation grid’s potential  $V_g$ , since it is small in comparison with  $V_a$ . The simplified CRT diagram was shown in [Fig. 5.1.12](#), which we repeat here for convenience:



**Fig. A5.1.1** (same as [Fig. 5.1.12](#)): The CRT’s ‘electron gun’ is formed by the hot cathode, the intensity modulation grid (negatively charged by  $V_g$ ), and the anode (positively charged by  $V_a$ ). The electrons emitted by the cathode are accelerated by the field  $E = V_a/d_{ka}$  and some of them pass through a small aperture in the anode. Within the deflection field they are additionally accelerated in the direction perpendicular to their motion. By hitting the phosphorescent coating of the screen the material is activated, releasing the excess energy as light. If the CRT deflection plates are made in sections (usually between 4 and 8), connected by a series of T-coil peaking circuits, the amplifier is loaded by a much smaller capacitance, so the system’s cut off frequency is several times higher. Also, the T-coils provide the time delay necessary to keep the deflecting field (as seen by the electrons in the writing beam) almost constant throughout the electron’s travel time. For simplicity only the vertical deflection system is shown here, but a similar circuit can be used for the horizontal deflection, too (as was in the Tektronix 7104 model 1 GHz oscilloscope). Note that owing to the increasing distance between the plates their length and width should also vary accordingly, in order to compensate for the otherwise reduced capacitance.

If for simplicity we assume a homogeneous electric field between the anode and cathode, the electrons are accelerated at a constant rate. Then the final electron velocity just before hitting the anode or passing through a small aperture in it, can be calculated by this relation, derived by considering the ‘classical’ laws of physics:

$$v = \sqrt{\frac{2 q_e V_a}{m_e}} \quad (\text{A5.1.2})$$

However, this relation can be considered correct only if the final velocity is small in comparison with the propagation velocity of electromagnetic fields (or light); otherwise, we must apply the Theory of Relativity. With  $V_a = 100 \text{ V}$  the above equation gives a final velocity  $v = 5.93 \times 10^6 \text{ m/s}$ , which is almost 2% of the velocity of light ( $c = 299\,792\,458 \text{ m/s} \approx 3 \times 10^8 \text{ m/s}$ ). Usually CRT anode voltages range from 25 kV to 35 kV, resulting in final electron velocities close to  $c/3$ , therefore the ‘relativistic’ effects can not be neglected.

The Theory of Relativity, as formulated by *Albert Einstein* (1879–1955) in 1905, predicts an increase of mass of an accelerated object in comparison to its rest mass, using the Lorentz transform (after *Hendrik Lorentz*, 1853–1928):

$$m_r = m_0 \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} \quad (\text{A5.1.3})$$

where  $m_r$  is the object’s relativistic mass,  $m_0$  is the object’s rest mass,  $v$  is the object’s velocity and  $c$  is the velocity of light. According to this, the electron mass changes with  $v$ , but  $v$  also changes owing to the acceleration enforced by the electric field, so both the final velocity and the relativistic mass must be calculated iteratively or by integrating a differential equation.

Once the accelerated electron passes through the anode’s aperture its velocity towards the screen remains nearly constant until it enters the deflection field. Then only its transversal velocity component changes and its path becomes parabolic. Once out of the deflection field the electron path is linear again (at least in CRTs which do not have additional focusing and trace rotation correction), until it hits the screen. The effective angle of deflection is proportional to the arctangent of the ratio of the transversal (deflection) velocity to the longitudinal (anode to screen) velocity:

$$\alpha = \arctan \frac{v_T}{v_L} \quad (\text{A5.1.4})$$

In comparison with the anode voltage the deflection voltage is low (usually  $< 100 \text{ V}$ , and even  $< 10 \text{ V}$  for highly sensitive tubes). So to calculate the amount of deflection we can use [Eq. A5.1.2](#) (using, of course, the instantaneous deflection voltage  $V_d$  instead of the anode voltage  $V_a$ )<sup>1</sup>; however, instead of the rest electron mass  $m_e$ , the relativistic mass  $m_r$  must now be used.

<sup>1</sup> Note that in order to avoid confusing velocities and voltages we here use the symbol ‘ $v$ ’ for velocity and ‘ $V$ ’ for voltage, regardless of whether either the voltages and/or velocities are constant or they change with time (with the exception of the propagation velocity of EM waves in free space, which is a universal constant and thus denoted by  $c$ , as usual).

The attempt to find the electron's final velocity analytically would take us too far from the problem we are trying to explain, so let us be satisfied with the approximation  $v_L = c/3$  for the anode voltage  $V_a = 35$  kV. In this case the electron would, upon acceleration, acquire a relativistic mass  $m_r \approx 1.06 m_e$ . By using  $m_r$  in [Eq. A5.1.2](#) to calculate the vertical velocity component  $v_T$  after the deflection, we obtain a value some 3 % lower (compared to the value calculated with  $m_e$ ). To obtain the desired deflection we must increase the deflection voltage by the same amount. This error is constant and is corrected by increasing the amplifier's gain accordingly.

However, a more difficult problem arises from the electron taking a finite amount of time to travel the deflection field while this field is changing in the vertical direction with the input signal (or in the horizontal direction with the time base's voltage ramp). For a constant or slowly changing deflection field the travel time does not have to be accounted for; but when it changes by about 1 V/ns or more the travel time is significant, since both the horizontal and vertical deflection are affected by a different amount at different combinations of time bases and signal frequencies. The resulting error is impossible to compensate by a change in amplifier gain only.

If we again assume  $v = c/3$  and a deflection field length  $l_d = 5$  cm (in the anode to screen direction, not between the plates!), the electron would travel the deflection field distance in time:

$$t_d = \frac{l_d}{\frac{c}{3}} = \frac{5 \times 10^{-2} \text{ [m]}}{\frac{3 \times 10^8 \text{ [m/s]}}{3}} = 0.5 \text{ ns} \quad (\text{A5.1.5})$$

and this time is comparable to the risetime of a 700 MHz bandwidth oscilloscope. Therefore the amount of deflection in both the horizontal and vertical directions would depend also on the signal's slew rate or frequency and not only on its amplitude, as it should. The effective change in deflection sensitivity varies as:

$$\frac{V_{d\omega}}{V_{dDC}} = \frac{\sin(\omega t_d)}{\omega t_d} \quad (\text{A5.1.6})$$

where  $\omega$  is the signal frequency,  $V_{d\omega}$  is the peak signal deflection voltage,  $V_{dDC}$  is the DC deflection voltage giving the reference amount of deflection, and  $t_d$  is the transition time which the electron spends in the deflection field.

This problem is solved by making the deflection plates from a number of smaller sections (usually between 4 and 8), as shown in [Fig. A5.1.1](#). Between the sections, an appropriate signal time delay is introduced using T-coils (see the envelope delay characteristics of T-coils in [Sec. 2.4](#); the total delay of all sections should be as close as possible to  $t_d$ , [Eq. A5.1.5](#)). Thus, if the signal propagation delay is matched to the electron's velocity, the electron is being deflected by an almost constant field throughout its deflection field travel time. The resulting deflection is a function of only the instantaneous signal amplitude, independent of the slew rate and frequency.

An additional benefit of this technique is a lower effective capacitance of the deflection plates, as seen by the output transistors, reduced to only a fraction of the value obtained when the plates were made as a single block. The reduced capacitance helps us to extend the bandwidth by a very large factor.

Returning to our amplifier example in [Sec. 5.1](#), if we were to use a CRT with the deflection plates made of 4 sections and assuming the same total capacitance (all sections) of 9 pF, then each section would have only 2.25 pF. Each T-coil inductance should then also be lowered accordingly, increasing the cut off frequency. In turn, both  $L_c$  and  $C_c$  should also be reduced. So, initially, the collector of transistor  $Q_4$  needs to supply the current to the first section only, the following sections are supplied later, partly by  $Q_4$  and partly from previous sections. This means that, effectively,  $Q_4$  is loaded by only a fraction of the total capacitance.

A quick estimation shows that by recalculating all the inductances, starting from the reduced capacitance of a single section as a load of the last T-coil and still using the same value of the load resistor  $R_b$ , our amplifier would be almost three times faster (about 250 MHz, instead of just 88 MHz).

The readers who have followed the text and the equations in [Sec. 5.1](#) and their development in [Part 2](#), should now be able to execute the calculations of the peaking components for the new amplifier, so we offer this example as an exercise challenge.

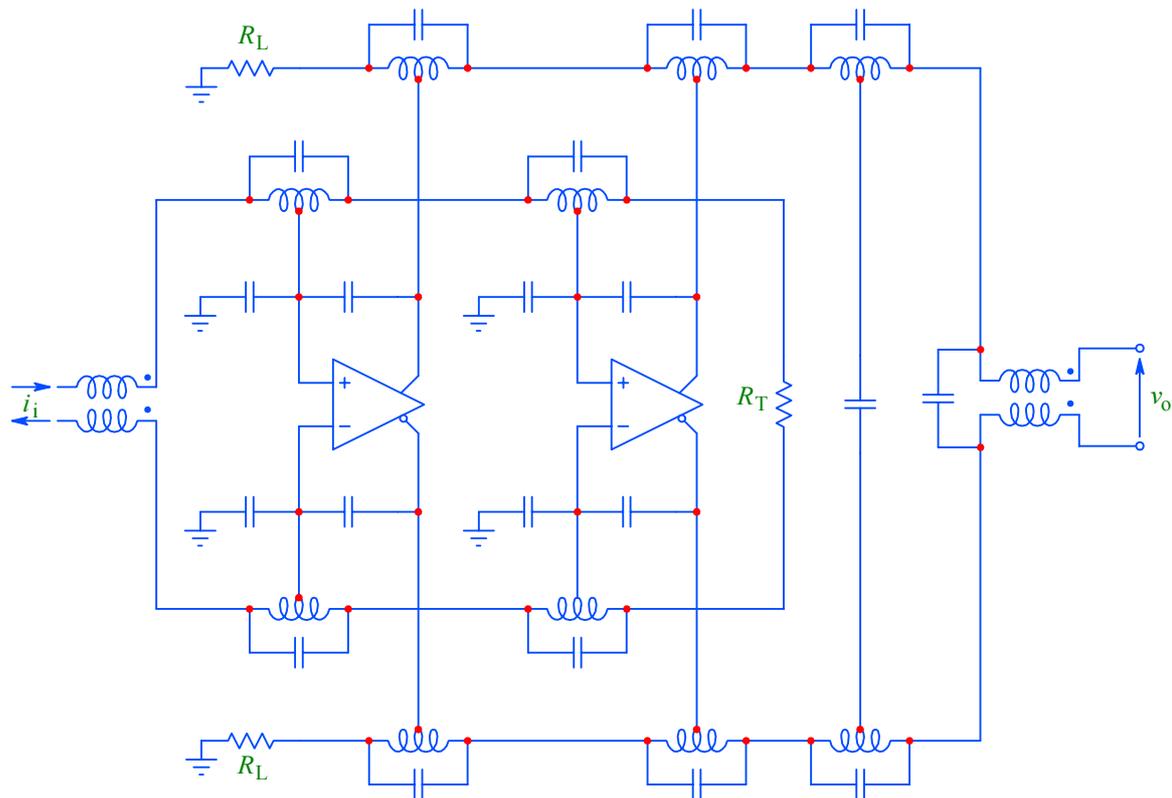
The history of Tektronix transistorized oscilloscopes, using this technology, starts in the early 1960s, when the 50 MHz 453 model was upgraded to the 150 MHz 454 model by employing a distributed deflection field of six sections. This was not the beginning, though: in the late 1950s *John Kobbe* developed and designed a balanced counter-wound delay line for the 580 series (which used vacuum tube amplifiers). Those delay lines were probably ‘accidental’ T-coil designs, since there was no bridge capacitor; instead, the parasitic capacitance between adjacent windings was doing the job. The 1 GHz 7104 'scope, designed by *J. Addis* and *W. Gross* in 1979, used delay lines in both vertical and horizontal deflection; consequently its XY performance was much, much better than anything before it.

Speaking of the 7104, it would be inappropriate to neglect the dramatic increase in performance resulting from a revolutionary CRT design by *Hans Springer*, described in his own article [[Ref. A5.1](#)]. Although several components were contributing to the overall performance improvement, such as the helicoidal delay line deflection plates for both X and Y axis, or the seven–element electric field lenses which increased the deflection by about 4.5 times, etc., the key component was the so called ‘micro-channel plate’ (MPC).

The MPC was placed some 3 mm from the screen phosphor coating and acted as the electron multiplier to compensate for the low beam current required to enhance the deflection sensitivity to 1 V/div. The plate had a large number of parallel ‘tunnels’ about 25  $\mu\text{m}$  in diameter, slightly at an angle to the electron beam. The tunnels were plated with a conductive material exhibiting secondary emission. Both sides of the plate were also metalized and DC biased by 700–1000 V. Thus when a single electron entered the tunnel and collided with the wall secondary electrons were generated and accelerated, colliding with the wall again to generate further electrons, and so on, until finally leaving the plate and hitting the phosphor coating. Further focusing was provided by an additional 10 kV potential between the MPC and the aluminized surface of the phosphor coating. The resulting spot size was about 0.4 mm in diameter. In this way a single nanosecond scale event, repeating only once or twice a second, could be clearly visible under normal ambient lighting conditions.

Science and technology normally progress in small steps, often a 20–30% improvement in performance is considered worthy of announcing. An occasional factor of 2 or 3 is a ‘great achievement’, whilst a rare factor of 10 is a ‘revolution’. In the case of the 7104 CRT its three-fold increase in speed, four-fold in sensitivity, and a 1000-fold in brightness results in an overall performance improvement of about 12 000, which is probably unique in modern history.

Speaking of the T-coils used as delay lines, we must mention another important wideband application, the so called ‘distributed amplifier’, [Fig. A51.1](#).



**Fig. A5.1.1:** Distributed amplifier principle, showing only two sections. The inner T-coils form the input differential delay line, the outer ones form the output differential delay line. The amplifier outputs are effectively added, so the system bandwidth can be extended far beyond the individual amplifier bandwidth by increasing the number of stages. In addition, the system gain can be greater than one, even if the individual stage gain is lower than one. Note also the input capacitance compensation by positive feedback (‘bootstrap’).

Such an amplifier consists of a number of equal amplifying stages, effectively connected in parallel by two delay lines, one on the input side and the other on the output side. Since the amplifier outputs are added and they all drive the same load, the system’s bandwidth can be increased well beyond the single stage cut off frequency.

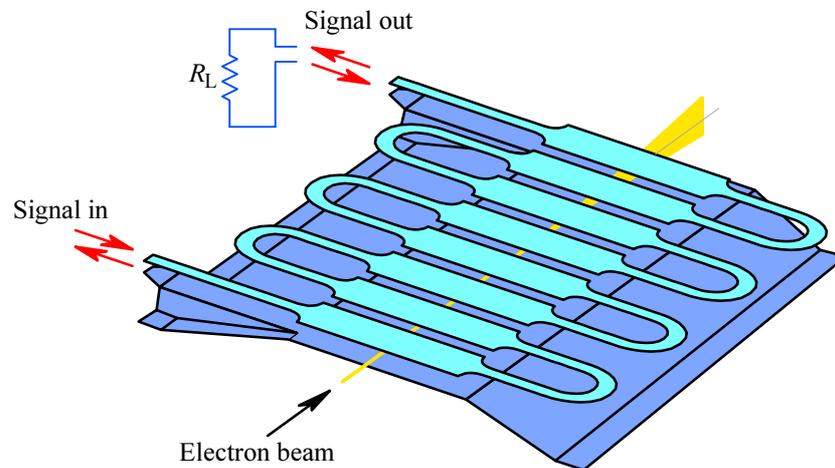
The history of distributed amplification can be traced back to a British patent by *W.S. Percival* in 1936, and the classic paper by *Ginzton, Hewlett, Jasberg and Noe*, published in the *Proceedings of the IRE* in August 1948.

Unfortunately, bipolar junction transistors are not suitable for distributed amplification, owing to their low and complex input impedance. Although the T-coil can be designed to partially compensate this for a single stage (as we have shown in [Part 3, Sec. 3.6](#)), the situation quickly degrades with increasing the number of stages.

Therefore only devices with a dominantly capacitive input are suitable for the task. Vacuum tubes were used in the past and MOSFETs are used in these days. However, it is very difficult to implement such systems in an integrated circuit (i.e., see [Ref. 5.63, 5.64 and 5.65]), so most distributed amplifiers are still designed with discrete devices.

In modern digitizing oscilloscopes the CRT bandwidth has lost its importance, and many CRTs have been replaced by LCD matrix displays. Nevertheless, T-coils can still be found, i.e., in the Tektronix 11A72 dual channel plug-in preamplifier hybrid (designed for the 1 GHz 11402 digitizing oscilloscope in 1988), in the form of a 1.3 mm spiral structure, having enough mutual inductance to cancel the inductance of the bonding wire in series with it. And if we could look under the cover of the latest chips we would probably find similar things.

An extreme example of the deflection system evolution is shown (simplified) in Fig. A51.2. The configuration was developed at Tektronix for their 1 GHz 519 model, designed by *Cliff Moulton* already in 1961.



**Fig. A5.1.2 :** The deflection plates developed at Tektronix for their 1 GHz CRT ('scope type 519, developed in 1961), where the plate geometry itself forms a delay line.

Here the deflection plates themselves form a delay line. A major design effort was needed to control the deflection field gradient, as well as the delay line impedance ( $125\ \Omega$ ) for correct delay time and low signal reflections. The delay line support screws (not shown here) served also the purpose of adjusting the shunt capacitance separately for each section.

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#### Reference:

- [A5.1] *H. Springer*, General Purpose Oscilloscope with GHz Bandpass, *Electronic Engineer*, Mid-May, 1979, pp.79–84