VOLUMETRIC COMPRESSION BEHAVIOR OF SAND UNDER CYCLIC LOADING HISTORY WITH VARIOUS STRESS PATHS

Toshiyasu UNNO¹, Motoki KAZAMA², Ryosuke UZUOKA³ and Noriaki SENTO⁴

ABSTRACT

The objective of this study is to compare the volumetric change of liquefied sand with that of dry sand under the same cyclic shear histories. To do this, a multiple series of strain-controlled cyclic shear triaxial tests was conducted under both undrained and drained conditions. Under the same cyclic shear strain loading histories, the test results illustrated that the volumetric strain of drained cyclic test of dry sand was the same as that of post-cyclic saturated sand. The volumetric change under cyclic loading depends only on the strain histories has no relation to the stress paths. In addition, it is found that the lower limit value of volumetric compression under cyclic shear can be determined. Under large shear, the void ratio of sandy material converged at a particular value.

Keywords: Liquefaction, cyclic shear, volumetric compression, void ratio, sand

INTRODUCTION

In order to predict the residual deformation during and after an earthquake, the cyclic and post-cyclic behaviors of sand have been studied by many researchers (e.g. Lee and Albas, 1974., Nagase and Ishihara, 1992., Shamoto, Sato and Zhang, 1996., Ishihara and Yoshimine, 1992.) in the laboratory. Volumetric compressibility after liquefaction is an especially important factor to understand the settlement of liquefied ground (e.g. Yoshimi and Oh-oka, 1976., Silver and Seed, 1971., Martin, Finn and H.B. Seed, 1975., Yoshimi and Tokimatsu, 1977.). Volumetric compressibility of saturated sand after liquefaction is expected to be related to that of dry sand during drained cyclic shear because the compression behavior due to cyclic shear is essentially the same except for the stress-strain paths. However, few attempts have been made to compare the volumetric compressibility under both testing conditions.

In this study, the authors compare the volumetric strain of saturated sand after undrained cyclic shear with that of dry sand during the same cyclic shear strain histories. In the former test, the amount of volume compression is not determined until drainage after undrained cyclic shear is completed. On the other hand, in the latter test, volumetric strain can be measured continuously during cyclic shear. Therefore, if there is a certain relationship between two, it will be possible to evaluate the amount of volume compression of saturated sand after undrained cyclic shear from the drained cyclic shear test of dry sand.

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TESTING SYSTEM AND TEST CONDITIONS

Test specimen
The testing material used in this study is fine clean silica sand known as Toyoura sand. Table 1 shows the physical properties of the sand. Cylindrical specimens of 5 cm in diameter and 10 cm in height are used in the triaxial test. Specimens with 26% and 60% of the initial relative density were prepared with the air-pluviation method.

<table>
<thead>
<tr>
<th>Specific gravity (G_s)</th>
<th>Minimum void ratio (e_{\min})</th>
<th>Maximum void ratio (e_{\max})</th>
<th>Minimum dry density (\rho_{\text{dry}}(g/cm^3))</th>
<th>Maximum dry density (\rho_{\text{dry}}(g/cm^3))</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.643</td>
<td>0.605</td>
<td>0.977</td>
<td>1.34</td>
<td>1.65</td>
</tr>
</tbody>
</table>

Strain-control cyclic shear triaxial tests
In order to study the volumetric compressibility of sand subjected to various cyclic shear histories, a multiple series of strain-controlled cyclic shear triaxial tests were conducted under the undrained condition and drained condition. Figure 1 shows the systematic diagram of the strain-controlled triaxial test. A step motor was used for cyclic loading, and the volumetric strain was measured directly with an inner cell, whose volumetric strain is measured not more than 25%.

Test conditions and test series
Table 2 shows the initial properties and test conditions. The initial mean effective principal stress for isotropic consolidation prior to undrained cyclic loading was 20 kPa. The maximum axial strain and cyclic number were selected as the parameter indicating the degree of cyclic loading history. Figure 2(a) shows the time history of axial strain during the cyclic loading process. A sinusoidal waveform (frequency of 0.005 Hz) was used for cyclic loading. The frequency was slow enough to achieve the equilibrium condition of volume change.

Laboratory tests were performed with two different approaches, as shown in Figure 2. One was a series of tests carried out under drained conditions using dry sand, with the volumetric strain measured continuously during the cyclic shear process (D-test). The other was that of an undrained-reconsolidation tests of saturated sand, with the volume change measured after undrained cyclic shear (U-R-test).

Figure 1. Test aspartame
Table 2. Initial test condition

<table>
<thead>
<tr>
<th>Sample</th>
<th>Test No</th>
<th>Specimen condition</th>
<th>Initial $\varepsilon$</th>
<th>Initial $\rho_{d}$ ($g/cm^3$)</th>
<th>Initial MEPS* $\sigma_{inj}$ (kPa)</th>
<th>Axial strain (%)</th>
<th>Drainage condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Toyoura sand</td>
<td>a-1</td>
<td>Dry</td>
<td>0.743</td>
<td>1.516</td>
<td>0.2, 0.4, 0.8, 1.2, 1.6, 2.0</td>
<td>D-test</td>
<td></td>
</tr>
<tr>
<td>D50=60%</td>
<td>b-2</td>
<td>Full saturated</td>
<td>0.749</td>
<td>1.511</td>
<td>0.2, 0.4</td>
<td>U-R-test</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b-3</td>
<td>Full saturated</td>
<td>0.742</td>
<td>1.517</td>
<td>0.2, 0.4, 0.8, 1.2</td>
<td>U-R-test</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b-4</td>
<td>Full saturated</td>
<td>0.733</td>
<td>1.525</td>
<td>0.2, 0.4, 0.8, 1.2, 1.6, 2.0</td>
<td>U-R-test</td>
<td></td>
</tr>
<tr>
<td>Toyoura sand</td>
<td>c-1</td>
<td>Dry</td>
<td>0.854</td>
<td>1.425</td>
<td>0.2, 0.4, 0.8, 1.2, 1.6, 2.0</td>
<td>D-test</td>
<td></td>
</tr>
<tr>
<td>D50=26%</td>
<td>d-2</td>
<td>Full saturated</td>
<td>0.852</td>
<td>1.427</td>
<td>0.2, 0.4</td>
<td>U-R-test</td>
<td></td>
</tr>
<tr>
<td></td>
<td>d-3</td>
<td>Full saturated</td>
<td>0.852</td>
<td>1.427</td>
<td>0.2, 0.4, 0.8, 1.2</td>
<td>U-R-test</td>
<td></td>
</tr>
<tr>
<td></td>
<td>d-4</td>
<td>Full saturated</td>
<td>0.814</td>
<td>1.457</td>
<td>0.2, 0.4, 0.8, 1.2, 1.6, 2.0</td>
<td>U-R-test</td>
<td></td>
</tr>
</tbody>
</table>

* MEPS: Mean effective principal stress

**Figure 2. Test procedures of D-test and U-R-test; (a) time history of axial strain, (b) test procedures**

**D-test (using dry sand specimen)**
In the cyclic loading process, the specimen was subjected to a step pattern strain-control cyclic shear history as shown in the Figure 2. The maximum axial strain was 2.0%.

**U-R-test (using full saturated sand specimen)**
In the cyclic loading process, the specimen was subjected to three loading patterns. The maximum axial strains of the three patterns were 0.4%, 1.2% and 2.0%, respectively as shown in Table 2. After the cyclic loading process, the volumetric strain was measured with the drainage of pore water. Here, we defined the volumetric strain as the drainage volume which was caused until the effective stress recovered to the initial effective stress (= 20 kPa).

**THE VOLUMETRIC COMPRESSION RELATIONSHIP IN DRAINED CYCLIC SHEAR AND RECONSOLIDATION AFTER UNDRAINED CYCLIC SHEAR**

**Test results**
**Stress-strain relationship**
Figures 3 and 4 illustrate the response time history of deviator stress during the cyclic strain loading of exemplary test results. Figures (a) and (b), respectively, indicate the results of the D-test, and the U-R-
test. It can be seen from the figures that the drainage condition during cyclic strain loading had a more significant influence on the deviator stress response rather than on the difference in initial density. That is, the deviator stress response increased with increasing axial strain during cyclic loading in the D-test, but in the U-R-test, the stress decreased dramatically in the first few cycles and then maintained an extremely small value. This can be attributed to the densification in the D-test and the liquefication in the U-R-test. The difference of the stress response behavior between dry sand and saturated sand was similar even though the initial density was different.

![Figure 3. Time history of deviator stress (Dr₀=60%); (a) dry sand (D-test), (b) saturated sand (U-R-test)](image)

![Figure 4. Time history of deviator stress (Dr₀=26%); (a) dry sand (D-test), (b) saturated sand (U-R-test)](image)

**Volume change during cyclic loading in D-test and reconsolidation after cyclic loading in U-D-test**

Figure 5(a) shows the time history of the void ratio change of dry sand during the cyclic shear process. In Figure 5(b), the void ratio was normalized by the initial void ratio. The void ratio decreased gradually with the number of cycles. The amount of change in the void ratio is much larger as the initial void ratio is larger. Figure 6 shows the void ratio change in the reconsolidation process after undrained cyclic shear. It can be seen that the void ratio decreased as the cyclic loading history increased.

In order to compare the volume compressibility due to cyclic loading in the two series of tests, we used the accumulated shear strain as an index to explain the cyclic shear history. The accumulated shear strain was defined by the following equation.

\[ \gamma_{\text{acc}} = \int |\dot{\gamma}(t)| dt \]  \hspace{2cm} (1)

Here, \( \dot{\gamma}(t) \) is the velocity of the shear strain at time \( t \).
Figure 5. Volume change of dry sand during the cyclic shear process (D-test); (a) Time history of void ratio, (b) Time history of normalized void ratio

Figure 6. The volume change of saturated sand during the reconsolidation process after cyclic shear versus mean effective stress (U-R-test); (a) $D_r = 60\%$, (b) $D_r = 26\%$

Figure 7. The comparison of volume change of dry sand during cyclic shear and that of saturated sand after undrained cyclic shear.

Figure 7 shows a comparison of the void ratio change between dry and saturated sand samples. The amount of volumetric compression of the D-test and the U-R-test is consistent irrespective of the deviator stress response shown in Figures 3 and 4. That is to say, the amount of volumetric compression depends on shear strain history, not on the drainage condition, or, in other words, the stress history. In addition, the initial relative density of the samples had no bearing on the results.
**Discussion**

Under the same cyclic shear strain loading histories, the results illustrated that the volumetric strain of dry sand in the drained cyclic test was the same as that of saturated sand in the post-cyclic reconsolidation test. The volumetric change under cyclic loading depends only on the strain histories and has no relation to the stress paths. Considering that volume change due to particle rearrangement is determined only by geometrical deformation, this conclusion seems acceptable. From the results, it can be concluded that a drained cyclic shear test can be used as an alternative to obtain the volumetric compressibility after liquefaction for saturated sand.

On the other hand, because the test results were obtained from limited test conditions, further study is necessary on the following points;

*a) Applicability to soils composed of crushable soil particles*

The sand samples used in this study were composed of relatively strong soil particles. Because volume compressibility is effected by particle crushability as a matter of course, the effects of crushability should be considered when evaluating the compressibility of crushable sand.

*b) Effect of confining pressure*

The initial effective stress used in this study was 20 kPa. It is known that the dilatancy behavior of sand is effected by the confining pressure, and the volumetric strain of high confining pressure cases is larger than that of the low confining pressure (e.g. Tatsuoka et al., 1981). Therefore, additional experiments used high confining pressure conditions are required.

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**MINIMUM VOID RATIO OF SAND AFTER DRAINED CYCLIC SHEAR**

**Test results**

It is thought that the amount of volumetric compression of sand during cyclic loading has a limit. The minimum void ratio after cyclic shear was studied under the test conditions shown in Table 3. The initial relative density of all specimens was 60%. Three sinusoidal waves with different shear strain single amplitudes (0.2%, 0.8% and 1.6%, respectively) were applied to the specimen. The number of wave cycles was applied from 800 to 2000.

Figure 8 shows the variation in void ratios during the cyclic shear process. In this figure, the results of reconsolidation after undrained cyclic shear is superimposed with a rectangular mark. It can be seen that under large shear strain history, the void ratio converged at a particular value of approximate 0.623. This value is not the same as the minimum void ratio of Toyoura sand (=0.609) obtained from standard test procedure (JGS standard, JGS 0161). It is thought that the volumetric compression mechanism under cyclic shear history and that of the test method for maximum and minimum densities of sands(JGS standard, JGS 0161), which is hammered, may be a fundamentally different thing. The void ratio changes of soil subjected to different single amplitudes of strain were plotted on a unique line.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Test No.</th>
<th>Initial void ratio (e_0)</th>
<th>Specimen condition</th>
<th>Wave</th>
<th>Axial strain single amplitude (%)</th>
<th>Cyclic number (N)</th>
<th>Accumulated shear strain during cyclic loading</th>
</tr>
</thead>
<tbody>
<tr>
<td>Toyoura Sand Dr(_c\geq 60)%</td>
<td>e-1</td>
<td>0.763</td>
<td>Dry</td>
<td>Strain-controlled Sinusoidal wave 0.005Hz</td>
<td>0.2</td>
<td>2000</td>
<td>30.7</td>
</tr>
<tr>
<td></td>
<td>e-2</td>
<td>0.759</td>
<td>Dry</td>
<td></td>
<td>0.8</td>
<td>1000</td>
<td>60.2</td>
</tr>
<tr>
<td></td>
<td>e-3</td>
<td>0.762</td>
<td>Dry</td>
<td></td>
<td>1.6</td>
<td>1000</td>
<td>118.2</td>
</tr>
<tr>
<td></td>
<td>f-4</td>
<td>0.759</td>
<td>Full saturated</td>
<td></td>
<td>1.6</td>
<td>800</td>
<td>99.6</td>
</tr>
<tr>
<td></td>
<td>f-5</td>
<td>0.760</td>
<td>Full saturated</td>
<td></td>
<td>1.6</td>
<td>1000</td>
<td>118.2</td>
</tr>
</tbody>
</table>
Modeling of volume change under cyclic loading

We used the results to make a model to represent the volume change due to cyclic shear loading based on the test results. The accumulated shear strain was used as an index to evaluate the degree of cyclic shear loading history. The following functions express the relationship between the accumulated shear strain and the void ratio during cyclic shear loading.

\[ e_{\text{cyc}} = e_0 + (e_0 - e_{\infty})e^{-a\gamma_{\text{acum}}}, \]  

\[ e_{\text{v}} = (e_0 - e_{\text{cyc}})\frac{V_s}{V_0} \]  

Here, \( e_{\text{cyc}} \) is the void ratio after cyclic loading, \( e_0 \) is the initial void ratio, \( a \) is the material constant, \( e_{\text{v}} \) is volumetric strain, \( V_0 \) is the initial volume of the specimen and \( V_s \) is the volume of the soil particle. \( e_{\infty} \) is the converged value of the void ratio.

Figure 9 shows the compression of the void ratio versus the accumulated shear strain during cyclic shear loading, using Equation (2). In this study, a material constant \( a \) of 0.097 was selected for equation (2). Here, it is thought that the material constant \( a \) could be a function of the confining pressure. The line plotted in the model was in good agreement with the test results.
It is well known that the volumetric strain due to cyclic shear decreases with increases in the initial density. Because it is thought that the converged minimum value of the void ratio is constant even when the initial void ratio is different, the volume change for sand with various initial void ratios can be plotted (see Figure 10(a)). In the figure, the material constant $a$ of Toyoura sand is assumed to be 0.097. Figure 10(b) is an example of application of this model to the test result c-1, a-1 and e-1. This figure shows if confining pressure is the same—even if initial void ratios differ,—this model can express the volume compression under cyclic shear history.

**CONCLUSION**

The authors compared the volumetric compression of dry sand during drained cyclic shear loading with that of saturated sand during the reconsolidation process after undrained cyclic shear under the same shear strain history. The conclusions of this study can be summarized as follows:

1) Under the same cyclic shear strain loading histories, the volume change of dry sand determined by a drained cyclic test was almost consistent with that of saturated sand determined by a post-cyclic drained test. In other words, the volume change under cyclic loading depends only on the strain histories and has no relation to the stress paths.
2) Repetitive the cyclic shear resulted in that the void ratio of sand converging at a certain value depends on the material. It was found that the void ratio variation with accumulated shear strain during cyclic shear was unique.
3) The void ratio variation of sand subjected to cyclic shear can be represented by an exponential function of the accumulated shear strain with the initial void ratio, converged void ratio and material constant.

**REFERENCES**