THE CONTRIBUTION OF SKIN FRICTION TO THE LATERAL RESISTANCE OF THE SOIL ON PILES/SHAFTS

Petros CHRISTOU¹, Mike McVAY², Marc HOIT³

ABSTRACT

The modeling of deep foundations includes the implementation of springs represented by nonlinear curves. The lateral response is simulated with P-Y curves, the axial with T-Z curves and the torsional with T-Θ curves. The springs are attached at the centerline of the pile/shaft and therefore the effect of skin friction at the pile/shaft – soil/rock interface is neglected when considering lateral loading. However the skin friction applied at the opposite faces of the shaft generates a moment couple at the centerline of the shaft. Therefore of interest is how much does the skin friction contributes to the mechanism of the lateral resistance of the soil. The T-Z curve presented here and also the two types of P-Y curves for the Florida limestone were developed at the University of Florida. The first type of the P-Y curves was back calculated neglecting any effects of the side friction to the lateral response of the limestone whereas the second type was calculated including the effects of side shear. In this paper the implementation of the contribution of the skin friction to the lateral resistance of the soil and its application to the commercial software FB-MultiPier is presented. Evident from the curves is that the two types of P-Y curves vary significantly and failure to include the side shear especially when the lateral loads are large, could lead to un-conservative estimation of the soil response. The results from the implementation of these curves are compared to experimental data and show very good correlation.

Keywords: Soil structure interaction, Pile foundations modeling

INTRODUCTION

The modeling of deep foundations includes the implementation of “Winkler” springs. Current commercial deep foundation analysis software, e.g. FB-MultiPier (BSI) and LPILE, employ nonlinear P-Y and T-Z curves (i.e. nonlinear lateral and axial springs) acting along the centerline of the pile/shafts length. Figure 1 shows the axial, lateral and torsional nonlinear springs as those are implemented in FB-MultiPier. These codes neglect any effect of side shear at the pile/shaft – soil/rock interface when considering lateral loading. The side shear, which always occurs with opposite signs due to shaft rotation, will result in variable moment or couple τ-D along the length of the pile/shaft, Figure 2. This couple may be significant, especially for large diameter piles/shafts when founded in a strong material i.e. rock. In the case of short shafts (e.g. L/D =1–3) during the application of lateral loading, the tip of the shaft will also undergo lateral translation and rotation. This will result in variable unit end bearing and tip shear (Figure 2) along with moments within the shafts. However, for long shafts, little if any lateral translation occurs at the shaft tip, resulting in uniform end bearing, and no tip shear. The work presented herein focuses on the latter case, i.e. the effects of side shear and the associated couple (τ-D) in long shafts as well as the P-Y resistance as a function of rock strength, qν.

¹ Associate Director, Department of Civil and Coastal Engineering, University of Florida, FL, USA, Email: pchri@ce.ufl.edu
² Professor, Department of Civil and Coastal Engineering, University of Florida, FL, USA. Email: mcm@ce.ufl.edu
³ Professor, Department of Civil and Coastal Engineering, University of Florida, FL, USA.
Normally the P-Y curves of the soil are back calculated from experimental data and they do not include any contribution of the side shear to the lateral response. Rather, the contribution is indirectly implied based on the experimental data. However the effect of the side shear can be calculated based on the equilibrium of forces on the stress block (pile/shaft element), Figure 3. Failure to include that, as results show, can lead to un-conservative results.

The curves that represent the axial response of the Florida limestone (T-Z curves) are presented. Also presented are two types of curves which were back calculated from experimental data to represent the lateral response (P-Y curves) of the same material. The experiments were preformed in a centrifuge using synthetic limestone (“Gatorock”) to assure material homogeneity (i.e. strength and stiffness) as well as ensuring that the material could be placed in the centrifuge container (i.e. eliminate cutting and fitting). The centrifuge was used in order to replicate the in-situ stresses that are extremely important in a stress dependent material (i.e. angle of internal friction of Florida limestone 25 ~ 35 degrees).

CONTRIBUTION OF SIDE SHEAR TO THE LATERAL RESPONSE OF ROCK

When a shaft is embedded in rock strata and it is laterally loaded then the lateral response of the rock strata at any elevation along the shaft length is a function of the lateral resistance of the soil and the side shear (skin friction) that is developed at the interface of the shaft and the rock. The back calculated P-Y curves are functions of the properties of the rock and they do not account for the contribution of the side shear. Figure 3 shows a free body diagram of an element of the shaft of length dz. Based on force equilibrium we can calculate the lateral response of the soil per shaft unit length either neglecting or including the contribution the side shear forces. If we neglect the contribution of side shear, T, in the moment equilibrium for the resulting shear, V at a cross-section we get:

\[ V = \frac{dM}{dz} \]  

(1)

As a result, from lateral force equilibrium, the soil lateral resistance, P, is found as:
\[ P = \frac{dV}{dz} = \frac{d^2 M}{dz^2} \quad (2) \]

If on the other hand the contribution of T is included in the calculation of the moment then the equilibrium equation results in:

\[ \frac{dM}{dz} = V + \frac{TD}{dz} \quad (3) \]

or

\[ \frac{dM}{dz} = V + M_s \quad (4) \]

where: \( M = \) moment on cross-section

\( M_s = \) moment per unit shaft length from the side shear force, T.

Again from horizontal force equilibrium:

\[ P = \frac{dV}{dz} \quad (5) \]

Solving equation 4 for V and substituting into equation 5, we can calculate the rock lateral resistance, P, as shown below:

\[ P = \frac{d^2 M}{dz^2} - \frac{d(M_s)}{dz} \quad (6) \]

![Figure 2: Field load transfer in soil/rock under lateral load or moment.](image)

As previously mentioned current commercial software packages neglect the contribution of the side shear in the lateral response mechanism of the foundation. Thus equation 2 is used. However, it is evident from equation 6 that there is a reduction in the lateral resistance, P, of the rock which is equal to \( \frac{d(M_s)}{dz} \). The reduction is caused by the skin friction in the pile/shaft interface. The latter suggests that for large diameter drilled shaft field tests in stiff rock, the back computed P-Y curve from equation 2 may be un-conservative.
Implementation of Moment Due to Side Shear, $M_s$

The contribution of the side shear, $T$, to the moment is a function of the shaft diameter, the $T$-$Z$ curve of the rock and the rotation of the shaft. When lateral loading is applied on a shaft it causes a rotation of the shaft at any given cross section which causes the two opposite faces of the shaft to shift vertically and therefore mobilize side shear as shown in Figure 2. Based on the rotation, the Moment/length resistance, $M_s$, may be computed at any cross-section. The value of $M_s$ is a function of the unit side shear at the periphery of the shaft, which varies around the shaft’s circumference as shown in Figure 4.
Consider the shaft loaded along the direction AC of Figure 4. The cross-section of the shaft is divided into 6 slices in every quarter. $R_i$ is the distance from the center of shaft to the center of slice $i$ on the surface of the shaft. For example, $R_1$, is the distance from the center of the shaft to the middle of slice 1. In actuality since the shaft is divided into pie sections all $R_{is}$ equal the radius of the shaft. Next, the arc length $DQ$ is referred to as $C_1$ where the subscript 1 refers to slice 1. Assuming all the points to the right of line BD (e.g. $P$, $Q$, etc.) are moving down while those on the left of BD are moving up, the surface forces on each slice may then be calculated. The value of shear stress, $\tau_i$, for each slice is found from the vertical displacement, $Z_i$, which is a function of the rotation, $\theta$, and the distance from the center of cross-section to the center of the slice, $R_i$. If $Z_1$ is the average axial displacement of slice 1 and $\tau_1$ is obtained from T-Z curve knowing $Z_1$ then the side shear force/unit length, $T_1$, acting on slice 1 is given by:

$$T_1 = \tau_1 C_1$$  \hspace{1cm} (7)

The moment per unit shaft length about O, $M_{s1}$, is found by multiplying $T_1$ by the distance to the cross-section centroid, $R_1$:

$$M_{s1} = T_1 R_1 = \tau_1 C_1 R_1$$  \hspace{1cm} (8)

The total moment per unit length may be found by summing the moments acting on all the slices:

$$M_s = \sum_{i=1}^{n} \tau_i C_i R_i$$  \hspace{1cm} (9)

where: $n$ = number of slices.

In the above estimation of $M_s$, it is assumed that the neutral axis (i.e., center of rotation) of the shaft remains at the center of the cross section. Although this assumption is not the case for large strains, the effect of changing the position of neutral axis was found to be small on the estimation of $P$. For instance, an analysis with FB-MultiPier, of a 9 ft (2.75 m) diameter shaft with an applied lateral load of 5000 kips (22240 kN) at 7 ft (2.15 m) above rock surface showed a maximum movement of the neutral axis of 10 in (0.25 m). Subsequent calculation of $M_s$ with the cross-section’s center of rotation at the centroid or at the neutral axis made only a 1% difference in the calculation of $P$ (force/length).

**AXIAL (T-Z) SOIL RESPONSE**

The normalized T-Z curve shown in Figure 5 represents the axial response for the Florida limestone (McVay et al., 2004). The curve was constructed based on experiments conducted on drilled shafts embedded in synthetic limestone for 3 different strengths: 10 tsf (960 kPa), 20 tsf (1920 kPa), and 40 tsf (3840 kPa). All of the tests were performed on 6 ft (1.8 m) diameter shafts embedded 18 ft (5.5 m) (L/D = 3) into the limestone.

**LATERAL P-Y SOIL RESPONSE**

Two types of P-Y curves were back computed during the experiments:

P-Y curves neglecting all side shear on the shafts (McVay et al., 2004)  
P-Y curves considering unit side shear on the shafts (McVay et al., 2004)
All back computed curves were obtained from the 12 lateral load tests performed in the centrifuge with diameters of 6 ft (1.8 m) and 9 ft (2.75 m), embedment (L/D) of 2, 3, and 4 and limestone strengths of 10 tsf (960 kPa) and 20 tsf (1920 kPa).

**Back Computed P-Y Curves Neglecting Side Friction**

Figure 6 shows the typical back computed P-Y curve neglecting the contribution of the side shear. The P values have been divided by $q_u D$ and the y values by $D$ to make the curves dimensionless.

![Figure 5: Normalized T-Z curves for synthetic rock.](image)

![Figure 6: Back calculated P-Y curves, neglecting shaft side shear.](image)
As part of the P-Y curve development/validation, the curves were adjusted, validated through software (e.g. FB-MultiPier, LPILE) and used to predict the moments, displacements, etc. for each experiment. The resulting deflections of the shaft were compared with the actual deflections obtained from different load cases. Figure 7 shows the comparison of deflections for the shaft described above, at lateral load of 1880 kips (8360 kN) and 2845 kips (12655 kN).

![Figure 7: Comparison of deflections (Test vs. FB-MultiPier). Side shear effects are neglected.](image)

**Back Computed P-Y Curves Corrected for Side Friction**

Figure 8 shows the typical back computed P-Y curve corrected for the contribution of the side shear. The P values have been divided by \( q_u D \) and the y values by D to make the curves dimensionless. Again the curves were validated through software and then used to predict the forces, displacements etc. for each experiment. The results were compared with the experimental results and are shown in Figure 9.

![Figure 8: Back calculated P-Y curves, corrected for side shear.](image)
As with the case of T-Z curves, it is common practice to normalize P-Y curves with $q_u D$ and $D$ as was done for both the inclusion and exclusion of side friction. However, the computed P-Y curves for rock exhibit significant variability when normalized with linear rock strength and shaft diameter values. After a number of trials, it was found that, the un-corrected, Figure 10, and the corrected curves, Figure 11, could be represented by a single trend-line if the P values are normalized with $q_u^{0.25} D^{0.9}$ and $q_u^{0.15} D^{0.85}$, respectively. The curves are valid for all the experimental results (6 ft (1.8 m) and 9 ft (2.75 m) diameter shafts, different rock strengths, etc.). Also the P-Y curves are unit dependent. That is, the rock unconfined compressive strength ($q_u$), the shaft diameter and rock’s lateral resistance, P, must be in ksf, ft and kips/ft, respectively. Also shown in Figures 10 and 11 are the graphs for soft clay for 20 ksf (960 kN) and 40 ksf (1920 kN). These graphs show the current practice in the absence of rock curve data.
EXAMPLE

The shaft in Figure 12 is a 9 ft (2.75 m) diameter drilled shaft which is embedded 20 ft (6 m) within a limestone with an unconfined compressive strength of 20 tsf (1920 kPa). The example is used to show the shears that are developed shears within the shaft. Rock P-Y resistance provided by Matlock (Matlock, 1970), and the proposed curves (Figures 10 and 11) with and without side friction was considered. The soil models are as follows:

- PPY: Parabolic, Matlock, $c_u = 20$ ksf (960 kPa) ($q_u/2$)
- LNC: Proposed, Figure 10, no side shear correction
- LC: Proposed, Figure 11, side shear correction

Figure 13 shows the shear diagrams of the shaft for the three different P-Y models as obtained from the numerical analysis. Based on that we can make some interesting observations:

- The diagrams from the first two models (parabolic P-Y and limestone not corrected for side shear) show very similar response.
- The shape of the shear diagram for all soil types along the length of the shaft is reasonable.
- The shear is constant (500 kips (2225 kN)) in the pile to the point that enters the rock.
- The shear distribution within the rock has a parabolic shape and shows a maximum of -474 kips (-2108 kN) for the parabolic P-Y and -326 kips (-1450 kN) not corrected for side shear at a depth of about 10.7 ft (3.25 m) below the rock surface.
- The shear diagram for the LC model has a significantly reduced shear distribution. The correction for side shear in the LC model reduced the moment associated with the P-Y resistance.
CONCLUSIONS

In commercial software packages, e.g., FB-MultiPier, LPILE, etc., drilled shaft axial skin friction $T$, is assumed to act along the centerline of the shaft. Therefore, in the case of lateral loading, the shear and moment in a segment of the shaft is assumed to be resisted by the soil/rock lateral resistance, $P$, with no lateral resistance contributed by $T$. However, the results of this study show that in the case of large shafts, or shafts founded in strong rock, the true side shear, $T$, acting on the wall of the shaft may develop a significant couple (Moment, $M_s$) depending on rotation, $\theta$, of the cross-section. Not
accounting for this effect, i.e. combined axial and lateral loading, may lead to over prediction and un-conservative lateral shaft response. The effect is magnified if the applied axial loads are small. This may have a significant effect on both the Moment, M, and the back computed P-Y curve for a laterally loaded shaft. From a designers’ stand point this finding is very important since it will dictate the design (i.e. steel reinforcement) of the pile/shaft.

RECOMMENDATIONS

The curves developed in this research and presented here are based on limited data, i.e., 2 rock strengths and 12 lateral tests. The results must be used with caution.

REFERENCES