SEISMIC BEHAVIOUR OF FLEXIBLE RETAINING STRUCTURES: NUMERICAL MODELLING VS. SIMPLIFIED APPROACHES

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ABSTRACT

The seismic response of a retaining wall is undoubtedly a complex issue that involves the dynamic interaction between the wall and the retained soil. The problem becomes more complicated if the wall is flexible, especially when constrained by anchors or tiebacks, and in the presence of significant pore-pressure increase. However, most of the modern seismic norms cope with the design of flexible retaining walls in a rather simplistic way, based mainly on limit-equilibrium methods. The present study tries to shed some light on this important issue. Our aim is the realistic numerical calculation of the earth pressures and of the seismic distress developed along the structure, especially in terms of displacements. Regarding numerical modeling, a finite-difference based computer code has been considered, while as far as the soil behavior is concerned, linear and nonlinear analyses are conducted. The role of parameters, like the wall flexibility, the compliance of the foundation soil, the type of anchoring, and the characteristics of the seismic excitation is examined thoroughly. Based on the results of the numerical dynamic analyses, we have also checked several simplified approaches for dealing with the problem at hand, stemming from the available analytical solutions in the linear case, pseudo-static approaches based on limit-equilibrium considerations, and simplified dynamic approaches based on suitable modifications of the Newmark method, in order to provide guidelines in a format useful for seismic norms.

Keywords: flexible retaining walls, seismic earth pressures, numerical analysis

INTRODUCTION

Despite their structural simplicity, retaining walls are in general complicated structures as the estimation of their distress and/or response under static and mainly under dynamic conditions is a complicated soil–structure interaction problem. Experimental studies and earthquake reality have shown that retaining walls, made up either of caisson gravity walls or of passively-anchored sheet-pile walls are quite vulnerable to strong seismic shaking, especially when saturated cohesionless soils in the backfill and foundation exhibit a strength degradation. However, it is obvious that the seismic response of a flexible retaining wall subjected to kinematic constraints (e.g. anchors) is completely different from the response of rigid gravity wall, the behavior of which is dominated mainly by the potential mechanisms of sliding or tilting.

Despite the importance of the seismic design of many retaining walls, and although their vulnerability was amply demonstrated in recent earthquakes (Nihonkai-chubu 1983, Kobe 1995, Kocaeli 1999), most of the seismic norms worldwide cope with the seismic design of retaining walls in a rather simplistic way. Based mainly on the over-simplified concept of pseudo-static earth pressures, norms are limited to an approximation of the supposed dynamic earth pressures, and thereafter, to a rough

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estimation of the developed shear forces and bending moments. Additionally, the seismic norms usually ignore the developed displacements which may be of great importance for the design. There exist two main reasons why the estimation of wall displacements is crucial. The first is related to the fact that in many cases a specific range of displacements may be regarded as accepted in order to achieve a balance between the desired levels of safety and economy, respectively. As we deal with a dynamic soil–structure interaction problem, the development of controlled displacements leads to substantially lower dynamic earth pressures, and subsequently to a cost effective design. The second reason is related to the fact that a wall usually retains not only soil layers, but structures or infrastructures as well. It is evident that some of them can not undergo differential displacements—settlements beyond a certain acceptable limit (PIANC 2001).

The present study tries to shed some light on this important issue. The aim is the realistic estimation of the developed displacements for flexible retaining walls. After an extensive literature review of the design methods, a simple analytical method was applied. The method, developed by Towhata & Islam in 1987, combines (a) the concept of limit equilibrium of the retained soil wedge with (b) the well-known methodology of accumulated displacement that has been proposed by Newmark in 1965 for a rigid body that slides on an inclined surface.

The results are very interesting and indicative of the importance of the wall displacements. It is realized that the estimation of the wall displacement is a complicated issue that certainly do not depend on a portion of the peak ground acceleration (as limit–equilibrium methods usually imply), a fact that undoubtedly should be taken somehow into consideration in the seismic design.

METHODS OF ANALYSIS AND DESIGN

General
The seismic analysis and design of retaining structures was based for many decades on the simple Mononobe–Okabe (M-O) method (Okabe 1926, Mononobe & Matsuo 1929), which actually comprises an extension of the static Coulomb theory. According to M-O method, the seismic earth pressures are taken into account as pseudo-static earth pressures that depend on a fraction of the expected peak ground acceleration. However, the pseudo-static concept has limitations that stem from the limitations of the Coulomb theory: only one soil layer can be present, seismic excitation is taken into account as a single constant parameter, pore pressure built-up can be considered in a simplistic way, and of course, soil deformations or displacements are ignored. Nevertheless, being the main representative of the limit-equilibrium methods, M-O method (and some of its variations) prevailed mainly due to its simplicity and the familiarity of the engineers with Coulomb theory. However, experiments were proving that the M-O method was accurate only when the wall movements (sliding, tilting, bending deformations, or any possible combination of them) were large enough to cause non-reversible plastic deformations in the soil. As the kinematic constraints imposed on some retaining systems (like basement walls, bridge abutments, anchors) deter the limit-equilibrium conditions, increased dynamic earth pressures were observed, that could not be predicted by the limit-equilibrium methods.

In the early ‘70s, the first analytical elasticity-based methods were developed (e.g. Wood 1975). The predicted dynamic pressures were almost three times more than the corresponding ones predicted by the limit-equilibrium methods. This fact, in combination to the absence of spectacular failures of retaining structures, led to the impression that elasticity-based methods are over-conservative and rather improper for practical use. For that reason these methods were passed over for a long period, and on the other hand the limit-equilibrium methods prevailed, being incorporated in the most seismic norms. More recently (in 1994 and 1997) Veletsos & Younan proved that the relatively high dynamic earth pressures of the elasticity-based methods up-to-date could be attributed to their main simplistic assumption that the wall is rigid and fixed at its base. They have drawn to that conclusion after having developed an analytical solution that takes into consideration both the wall flexibility and the potential compliance of the wall foundation. It was discovered that, for realistic values of wall flexibility and
rotational compliance at the wall base, the dynamic earth pressures are substantially lower than the corresponding pressures predicted for rigid fixed-base wall. It is evident however that the elasticity-based methods have certain limitations. Their main disadvantage is the assumption of homogeneous soil material that behaves linearly. Although the hysteretic behavior of the soil may be taken into account, a realistic simulation of a potential nonlinear behavior is rather impossible. As described hereafter, the efforts to take into account the nonlinear soil behavior (before reaching the limit-equilibrium conditions) are very few.

**Limit-equilibrium Methods**

As mentioned before, the M-O method is the prevailing method for the seismic design of retaining walls. The method is based on the limit-equilibrium of a soil wedge retained by the wall. The inertia forces developed on the wedge due to the seismic excitation are also included in the force equilibrium. These forces are easily calculated from the horizontal \( a_h \) and the vertical \( a_v \) seismic coefficients. The seismic coefficients comprise portions of the peak ground acceleration \( A_{\text{max}}(= a_{\text{max}}g, \text{where } g \text{ is the acceleration of gravity}) \), and, though they are usually calculated empirically, a value of \( a_h \) close to 65% of \( a_{\text{max}} \) is considered to be realistic. Note that the vertical acceleration \( A_v (= a_v g) \) is regarded trivial, and it is usually ignored in the design.

However, almost 50 years later than M-O method was developed, it was further simplified by Seed & Whitman (1970). As M-O method calculates the total force \( P_{AE} \), and as the static force \( P_A \) is already known by Rankine or Coulomb theory, a dynamic coefficient of active earth pressures \( \Delta K_{AE} \) can be easily calculated by the following equations:

\[
\begin{align*}
\Delta P_{AE} &= P_A - P_A \Rightarrow \Delta P_{AE} = P_{AE} - P_A \\
\Delta K_{AE} &= \frac{1}{2} \Delta K_{AE} H^2 \\
\Rightarrow \Delta K_{AE} &= \frac{2(P_{AE} - P_A)}{\gamma H^2} \tag{1}
\end{align*}
\]

Based on the previous expression, that coefficient is a function of the applied horizontal acceleration. For \( a_h < 0.4 \), Seed & Whitman proposed a linear relationship between \( \Delta K_{AE} \) and \( a_h \). Specifically:

\[
\Delta K_{AE} \approx \frac{3}{4} a_h \tag{2}
\]

Therefore, the normalized active force \( \Delta P'_{AE} \) is given by:

\[
\Delta P'_{AE} \left(= \frac{\Delta P_{AE}}{\alpha_h \gamma H^2} \right) \approx \frac{3}{8} \approx 0.4 \tag{3}
\]

Similarly, for the passive earth pressures \( \Delta K_{PE} \) can be easily calculated by the following equations:

\[
\begin{align*}
\Delta P_{PE} &= P_p - P_p \Rightarrow \Delta P_{PE} = P_{PE} + P_p \\
\Delta K_{PE} &= \frac{1}{2} \Delta K_{PE} H^2 \\
\Rightarrow \Delta K_{PE} &= \frac{2(P_{PE} + P_p)}{\gamma H^2} \tag{4}
\end{align*}
\]

\[
\Delta K_{PE} \approx \frac{17}{8} a_h \tag{5}
\]

and the normalized active force \( \Delta P'_{PE} \) is given by:

\[
\Delta P'_{PE} \left(= \frac{\Delta P_{PE}}{\alpha_h \gamma H^2} \right) \approx \frac{17}{16} \approx 1 \tag{6}
\]
Note that the above equations are valid for dry soil conditions. In the case that the retained soil is below water table the equations are modified depending on the soil permeability. In the special case of very high permeability (e.g. \( > 10^{-3} \) cm/s) the soil particles move independently of the water, so hydrodynamic forces should be taken into account as well.

**Elasticity-based methods**

The advanced elasticity-based method of Veletsos & Younan (1994 and 1997), as well as its numerical evaluation by Psarropoulos et al. (2005), aims to the estimation of the amplitude and the distribution of the dynamic earth pressures applied on flexible walls capable to rotate at their base due to a seismic excitation. The effect of the involved parameters was surveyed. Soil was considered to act as a homogeneous visco-elastic layer characterized by constant density and infinite extend at the horizontal direction. The base of the wall and the soil layer are excited by the same horizontal seismic motion. The parameters examined are the characteristics of the excitation and of the soil layer, in addition to the stiffness of the wall itself and of its rotational spring at its base. Harmonic as well as seismic excitations were considered. Emphasis was put on the long-period, quasi-static excitations. The response of the dynamically excited system was expressed as the product of the corresponding quasi-static response with an appropriate amplification factor.

According to Psarropoulos et al. (2005), in the case of a rigid fixed-base wall the normalized active force \( \Delta P_{AE} \approx 1 \). That value is in accord with the corresponding value proposed by Wood’s solution (1975), but it is 2.5 times higher than the corresponding value proposed by Seed and Whitman (i.e. \( \Delta P_{AE} \approx 0.4 \)). On the contrary, in the case of a wall very flexible and almost free to rotate at its base, Psarropoulos et al. (2005) calculated \( \Delta P_{AE} \approx 0.4 \), a value that is in agreement with Seed and Whitman, but it is in contrast to Wood’s solution (i.e. \( \Delta P_{AE} \approx 1 \)).

**Inelastic methods**

As the above conclusions refer to the estimation of dynamic earth pressures, they are valid not only for flexible walls, but for gravity walls, as well. Regarding the latter, Richards & Elms (1979) proposed the design of them for acceleration levels lower than the expected ones. Naturally, that proposal implies sliding of the wall, and the response is satisfactory only when the wall accumulated displacement (sliding) is below an acceptable level. Richards & Elms describe a procedure for the correlation between the accepted sliding and the design acceleration. Al-Homoud & Whitman (1999), raising questions about the uniform acceleration field in the retained soil, tried numerically to take into account the soil amplification phenomena.

Quite recently many efforts to improve the design methods for flexible retaining walls have been made. Many of them, based on experimental data and real case studies, try to estimate the developed deformations and to fill actually the gap between limit-equilibrium and elasticity.

For example, Neelakantan et al. (1992) developed a simplified method of seismic design of flexible retaining walls, and they verified their method with shaking-table-test data. Their method defines a factor of safety against failure through the ratio between the active and the passive dynamic earth pressures. The method was then extended to anchored walls. Based on the experimental data, they claim that the proposed method improves the seismic design of flexible retaining walls in comparison to M-O method.

Steedman & Zeng (1990) performed centrifuge tests and they developed an analytical method of design of anchored flexible walls. They examined the effect of parameters, like the soil inhomogeneity with depth, the soil amplification, the hydrodynamic pressures, and the eigen-frequency of the wall. The results of their analyses are in accord to the M-O method. It was discovered however that the response of a flexible retaining wall depends significantly on its eigen-frequency.

In a modification of the “standard of practice” pseudo-static design method, so as to include deformation based analysis, Dennehy (1985) and Gazetas et al. (1990), developed an empirical relationship between sheet pile damage and sheet pile wall geometry for 75 seismic case histories in
Japan that were reported as not experiencing liquefaction flow failures during the specific earthquake. Note that the criteria for the occurrence of liquefaction is based on surface evidence (whereas the lateral displacements of retaining structures masks the evidence of liquefaction at depth), it is therefore questionable whether liquefaction did not occur at all of the 75 case histories. Two non-dimensional factors, the effective anchor index (EAI) and the embedment participation index (EPI) have been related to the degree of damage recorded. EAI is used to quantify the amount of available anchor capacity, while EPI provides the contribution of the wall embedment. Although this chart has greatly enhanced the deformation based design of sheet pile walls, limitations are noted: (a) there is no direct contribution from the wall stiffness or earthquake characteristics (i.e. intensity, frequency, duration) to the deformations, and (b) the EAI and EPI factors are indirectly a function of the seismic coefficients used in design, which has been noted by Kitajima and Uwabe (1979) to be unsatisfactory in relation to seismically induced deformations.

**Method by Towhata & Islam (1987)**

Towhata & Islam (1987), taking advantage of the simplified expressions given by Seed & Whitman (equations 2 and 5), developed a method of estimating the critical seismic coefficient necessary to initiate displacements of a soil wedge behind an anchored wall (Figure 1). In this way, the wall deformation can be estimated depending on the applied acceleration time history. Their approach is similar to the method developed by Newmark (1965) to estimate the accumulated displacement of a rigid block sliding on an inclined plane. In the case of dry soil, the equilibrium of forces applied on the wedge leads to:

\[
a_{sh} = \frac{a \tan \theta + \tan(\phi - \theta)}{1 + c \tan \theta}
\]

where:

\[
a = \frac{2(P_p + T_s)}{L^2 \gamma}
\]

\[
c = \frac{2}{L^2 \gamma} \left( \frac{17}{8} \frac{P_p}{K_p} + \frac{23T_s}{8(K_p - K_{sh})} \right)
\]

\[
P_p = \frac{1}{2} \gamma D^2 K_p
\]

\[
T_s = \frac{1}{2} h_i^2 (K_p - K_{sh})
\]

\[
K_p = \tan^2(45^\circ + \frac{\phi}{2})
\]

\[
K_{sh} = \tan^2(45^\circ - \frac{\phi}{2})
\]

where \(\phi\) is the angle of friction of the soil. The critical angle of sliding \(\theta_c\) can be calculated easily from the derivative of parameter \(a_{sh}\), and then the critical acceleration \(a_c\) can be evaluated. Then, the methodology of Newmark is applied for the estimation of the accumulated sliding. The proposed methodology is capable to estimate realistically the deformations developed on an anchored wall, taking into account the characteristics of the excitation.
A numerical application of the Towhata & Islam method was performed for a typical flexible wall with free height $H = 5\text{m}$, which retains (with or without anchoring) a soil layer with $\gamma = 20\text{kN/m}^3$ and $\phi = 32^\circ$. Only the special case of dry conditions was examined here. The main parameters of the problem were the embedment height $D$ (which was varying between 3 and 8 m), and the direction of the applied seismic excitations. Two accelerograms have been selected as seismic excitations. Both of them come from real seismic events in Greece. The first is the one and only accelerogram of the $M_S = 6.2$ Aegion earthquake in 1995, and the second is the Monastiraki record (MNSA) of the $M_S = 5.9$ Athens (Parnitha) earthquake in 1999. The acceleration time-histories, and the corresponding (acceleration $SA$ and velocity $SV$) response spectra of the two accelerograms are given in Figure 2. The selection of these two records was made on purpose, as a substantial discrepancy between their spectral characteristics exists, although both are characterized by comparable peak ground acceleration PGA, in the order of $0.5g$.

It is noted that most of the seismic norms worldwide regard PGA as the main design parameter. And this is certainly the case especially for the seismic design of retaining walls due to the pseudo-static concept (e.g. M-O method) usually adopted in the norms. Therefore, according to the norms these two excitations could or should be considered to be of similar importance. However, the results of this limited parametric study are indicative of the sensitivity of the problem to the spectral characteristics of the excitation (and not to the PGA). As shown in Figure 3, in the case of MNSA record (1999 Athens earthquake) the maximum wall displacement $d_{\text{max}}$ is of the order of 1.5 cm, while in the case of 1995 Aegion earthquake the corresponding $d_{\text{max}}$ is approaching 7 cm. This phenomenon can be mainly attributed to the significant differences of the spectral characteristics of the two excitations. Note that for $D = 8\text{m}$ the passive resistance is extremely high, leading thus to zero or negligible displacements.
Figure 3. Wall displacements calculated for the various cases examined. Note that in the case of MNSA record (1999 Athens earthquake) the maximum wall displacement \(d_{\text{max}}\) is of the order of 1.5 cm, while in the case of 1995 Aegion earthquake \(d_{\text{max}}\) is approaching 7 cm.

NUMERICAL VERIFICATION

One of the basic assumptions of the method developed by Towhata & Islam is that the passive resistance \(P_P\) originates from the whole wall embedment depth \(D\) (see equation 10). That assumption stems from the hypothesis that only translational movement of the wall is allowed. As in reality the wall is expected to rotate around a certain point at an active depth \(D'\) (smaller than \(D\)), this simplification of the method may lead to an overestimation of the passive resistance, and consequently to a non conservative design. In order to validate the accuracy of the Towhata & Islam method and to focus on a more realistic consideration of the passive resistance, a numerical study was performed utilizing the numerical finite-difference code FLAC (Itasca Consulting Group, 2005). FLAC is a 2-D program capable of modeling both static and dynamic situations. In this study an effective stress Mohr-Coulomb (M-C) constitutive model was used. The constitutive model is able to model plastic deformations utilizing a plastic flow rule. The elastic behavior of the soil is defined by the bulk and shear modulus, and the strength is defined by the angle of friction and cohesion. This significantly simplifies the dynamic soil behavior and is not capable of accounting for the strain dependent dynamic properties such as damping and shear modulus. Despite the use of this simplified constitutive soil model, it has been demonstrated by various researchers to yield satisfactory displacement results for a variety of applications. A sketch of the model and the corresponding finite-difference grid developed are shown in Figure 4. Note that wall is not anchored in that case.

Since in the numerical model the wall embedment \(D\) is considered to be 8m, the MNSA record (from Athens earthquake) is not considered, as it leads to no displacement (according to Towhata & Islam method). Therefore one more record characterized by high PGA (≈ 0.7g), namely the Gemona record from the \((M_s = 6.5)\) Friuli earthquake in Italy (1976), has also been used as base excitation.
Figure 4. Sketch of the numerical model and the corresponding finite-difference grid developed in FLAC. Free-field conditions (f.f.) are applied at the vertical boundaries of the model, while the seismic input motion is applied at the rigid base.

The cases examined are two. In the first case the wall embedment $D$ is considered to be 8m in both the analytical and the numerical model. So, the results of the Towhata & Islam method are directly compared with the results of the numerical model. As shown in Figure 5(a), the critical acceleration $A_y$ in the first case is very high ($\approx 0.5g$), leading thus to small wall displacements. For the Aegion excitation (with PGA = 0.54g) the maximum permanent displacement $d_{max}$ is only 0.02cm, while for Gemona (with PGA = 0.72g) the maximum displacement is around 0.2cm (Figure 5(b)). Note that the corresponding displacements predicted by the numerical analyses are around 3cm (Aegion) and 7cm (Gemona).

In the second case, it is assumed (hypothetically) that in the Towhata & Islam model the active wall embedment that causes passive earth resistance is $D' = 4m$ (i.e. at the middle depth of the wall embedment $D$). As shown in Figure 6(a), the critical acceleration $A'_y$ is decreased ($\approx 0.3g$), and, as it was expected, the wall displacements are much higher. For the Aegion excitation the maximum permanent displacement $d_{max}$ is 3cm, while for Gemona the maximum displacement is around 5.5cm, values comparable to the numerical results with $D = 8m$ (3cm and 7cm, respectively).
Figure 5. Case of full embedment ($D = 8m$): (a) acceleration time histories of the two excitations (Gemona and Aegion) in comparison to the critical acceleration $A_y$, and (b) the permanent displacement $d_{\text{max}}$ calculated. Note that $d_{\text{max}}$ is hardly of the order of 0.2 cm.

Figure 6. Case of reduced active embedment ($D' = 4m$): (a) acceleration time histories of the two excitations (Gemona and Aegion) in comparison to the critical acceleration $A'_y$, and (b) the permanent displacement $d_{\text{max}}$ calculated. Note that $d_{\text{max}}$ is between 3 and 6 cm.
CONCLUSIONS AND DISCUSSION

The present study was involved with the seismic design of flexible retaining walls, putting special emphasis on the estimation of displacements. After an extensive review of the available methods of analysis and design, a simple analytical method was applied. The method, developed by Towhata & Islam in 1987, combines (a) the concept of limit equilibrium of the retained soil wedge with (b) the well-known methodology of accumulated displacement that has been proposed by Newmark in 1965 for a rigid body that slides on an inclined surface. Two seismic excitations have been used, while the role of the involved parameters was examined. Then, in order to examine the relative accuracy of the simple analytical method, a series of finite-difference analyses has been performed with one more excitation. Judging of the results of the current study, the following conclusions may be drawn:

a) peak ground acceleration seems to have a minor impact on the wall displacements, while on the other hand, the fundamental period of the excitation (that actually represents the applied ground velocity) seems to play a very significant role on the response.

b) the analytical work of Towhata and Islam seems to overestimate substantially the passive resistance. However, we have to mention that extensive studies in the literature have shown that the method may lead to very large displacements, especially in the case of saturated backfills. Therefore, when this simple methodology is implemented, these facts should be taken somehow into account.

The final conclusion is that, as the developed displacements of a retained structure may be in general considerable, their reasonable estimation is an issue that has to be taken seriously into account in the design.

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REFERENCES


