CALIBRATION AND VERIFICATION
OF NONLINEAR WAVE PROPAGATION METHOD

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ABSTRACT

For the nonlinear inelastic response analysis of layered soils a new constitutive model has been developed by Gerolymos & Gazetas (Soils & Foundations, June 2005) as an extension of the Bouc–Wen model. Capable of reproducing the hysteretic behaviour of a variety of soils, the model possesses considerable flexibility to represent complex patterns of cyclic behaviour, such as stiffness decay and loss of strength due to build-up of pore-water pressure, cyclic mobility and load-induced anisotropy. After a brief highlight of the model, the paper outlines the calibration of the model parameters and the implementation of the model in the numerical wave-propagation analysis code NL-DYAS. Verification of the model and the code is made through comparisons with experimental results from the literature (centrifuge tests). For strong seismic shaking the inelastic wave propagation analysis reveals certain advantages of the method over the equivalent–linear method.

Keywords: nonlinear analysis, numerical modeling, calibration, experimental results

INTRODUCTION

Site response analysis is the first step in computing the structural response in a soil–structure system. Thus, thanks to the extended research on the issue during the last decades, a large variety of techniques is available for the geotechnical engineer to evaluate the response of soil deposits to strong earthquake shaking. The one-dimensional methods are the most commonly used, not only because of their simplicity, but also because it is often quite reasonable to assume the soil layers being almost horizontal and SH waves dominating in the wave field.

Equivalent linear analyses are the most popular owing to their computational convenience and simplicity. Their main limitations include their inability to efficiently predict the behavior of a nonlinear system especially under strong ground motion (Constantopoulos et al., 1973) and the violation of the principle of physical causality (Ching and Glaser, 2001).

In order to overcome these limitations several researchers have developed a number of nonlinear models (e.g.: Lee and Finn, 1978; Borja and Wu, 1994; Hashash and Park, 2001). However many of the commercially available models are not only difficult to be calibrated but are also incapable of predicting at the same time the observed shear modulus degradation and the damping increase for a given soil, and they usually overestimate damping at large strains.

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This paper illustrates the calibration and verification of a recently developed phenomenological nonlinear model (Gerolymos and Gazetas, 2005) which is capable of simulating even the most complex nonlinear characteristics avoiding the aforementioned disadvantages.

PROPOSED MODEL OUTLINE

Gerolymos and Gazetas (2005) developed a new phenomenological constitutive model (BWGG) as a modification of the original Bouc–Wen model (Bouc, 1971; Wen, 1976). The full BWGG model is very versatile and can reproduce quite complicated nonlinear behaviors such as the cyclic mobility and liquefaction. Having properly adjusted the parameters, the model predicts simultaneously any pair of shear modulus and damping curves with shear strain. Herein, only a brief outline of the model is given, omitting its capability to simulate strength deterioration, cyclic mobility and liquefaction.

According to BWGG model, the shear stress – strain relation is given as:

\[
\tau(t) = a \cdot G_{max} \cdot \gamma(t) + (1 - a) \cdot G_{max} \cdot \gamma_{y} \cdot \zeta \cdot \gamma(t)
\]  

(1)

where \( G_{max} \) is the small amplitude shear modulus, \( a \) is a parameter controls the post yielding stiffness, \( \gamma_{y} \) is a reference value of shear strain linked with the initiation of yielding in the soil. The parameter \( \zeta \) is the dimensionless hysteretic parameter that controls the nonlinear response of soil. This hysteretic quantity is among others a function of parameters \( A, n, b, g, s, s1 \) that control the shape of the hysteretic loop. Parameter \( A \) is proven to be equal to 1. Parameter \( n \) controls the rate of transition from the elastic to the plastic domain. Large values of \( n \) (greater than 10) model approximately a bilinear hysteretic curve; decreasing values of \( n \) lead to smoother transitions from the elastic to plastic region. Parameters \( b \) and \( g \) control the unloading–reloading branches of the hysteretic loop and their sum equals to 1. Parameter \( s \) controls the reversal stiffness and \( s1 \) is the initial value of the strain “ductility” which is defined in terms of current strain and the strain immediately before the last load reversal. The complete expression of \( \zeta \) and the detailed description of its parameters are given in Gerolymos and Gazetas (2005).

The BWGG model is implemented into a computer code named NL-DYAS which uses the explicit finite-difference technique to integrate the equations of motion for the nonlinear one-dimensional ground response analysis of layered sites.

Object of this paper is to present calibrated values of the parameters that can be used directly to perform a nonlinear site response analysis and verify through comparison with experimental data the efficiency of the BWGG model and NL-DYAS code.

PARAMETERS CALIBRATION

The most commonly available geotechnical data for a site response analysis include the profile of soil strength with depth of the deposit, the shear wave velocity profile, and the evolution curves of the shear modulus and damping with the shear strain. In this paper the well known \( G-\gamma, \xi-\gamma \) curves of the literature are used to calibrate the parameters \( n, b, s, s1, \) and \( \gamma_{y} \).

Using the method of the successive forward simulations (Santamarina and Fratta, 1998) an optimization procedure is implemented to determine the parameters to match the published experimental soil behavior by reducing the error between calculated and given data.

In Fig. 1 and 2 the modulus and damping curves predicted by the BWGG model are compared to the curves proposed by Ishibashi and Zhang (1993) for different values of plasticity index and
confinement pressure. The agreement of the calculated curves to the published ones is quite good in both stiffness and damping terms. The values of the parameters produce these results are summarized in Table 1. Parameter $b$ is fixed to 0.6 so that the predicted hysteretic loops comprise to observed experimental loops.

The parameters are also calibrated to match Vucetic and Dobry (1991), and Darendeli (2001) curves but the results are not presented here due to lack of space. The same procedure can be followed for any experimentally derived pair of modulus decline and damping increase curves.

![Figure 1. Comparison of published and calculated shear modulus and damping curves for PI = 0 and various confinement pressure levels](image1)

![Figure 2. Comparison of published and calculated shear modulus and damping curves for $\sigma' = 100$ kPa and various plasticity indices](image2)

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**Table 1. Calibrated model parameter values for Ishibashi & Zhang (1993) curves**
VALIDATION USING EXPERIMENTAL RESULTS

The efficiency of both model and code, and the validity of the calibrated parameters are examined through comparison with experimental results.

Centrifuge soil testing has been proven during the past decades to be an insightful method for understanding mechanisms governing the soil behavior and an extremely valuable tool for calibration and verification of analytical / numerical models. Of course, in the limited soil domain tested in the centrifuge, it is likely for 2-D effects to be developed. To diminish these effects laminar soil containers are used nowadays in most centrifuge tests. Even with some limitations centrifuge tests remain the most accurate experiments and thus useful tools for researchers.

A series of such tests have been conducted in the centrifuge of the University of California at Davis to investigate the nonlinear soil response in seismic excitations (Stevens et al., 1999). The results of the experiment DKS02 are used in this paper to verify the BWGG model and its calibrated parameters.

According to Stevens et al. (1999) a soil deposit consisted of a 0.6-m thick layer of dry Nevada sand with a relative density of 100 % and a unit weight of 16.8 kN/m\(^3\) was tested in the centrifuge with radial acceleration 10, 20, and 40 g simulating three deposits of the same soil material but of different depth: 6, 12, and 18 m. Through the measurement of the time delay in the arrival of a traveling shear wave caused by a hammer shock at the base of the deposit, the shear wave velocity was estimated to vary with depth with the law:

\[
V_s(z) = 165 \cdot z^{0.25}
\]

where \(V_s\) is the shear wave velocity in m/s and \(z\) is the depth in meters. The deposit is excited by the Santa Cruz record of the 1989 Loma Prieta earthquake with a maximum acceleration of 0.4 g.

Given the above data and using the BWGG parameter values estimated for sand behaving according to Ishibashi and Zhang curves for PI = 0, the centrifuge experiment is analyzed with NL-DYAS code. The same analysis was conducted using the equivalent linear program SHAKE (Schnabel et al., 1972). The results are displayed in Figs. 3 and 4 for the case of centrifuge radial acceleration 10 g.

In terms of surface acceleration time histories (Fig. 3) a fair agreement is noticed between the measured and the calculated by NL-DYAS time history. On the contrary, SHAKE seems to overestimate the acceleration at surface level. The phenomenon is more evident in Fig.4 where the response spectra at the surface are illustrated. SHAKE tends to predict not only higher PGA but also higher spectral acceleration up to a period of 0.12 sec. However, for higher periods the spectral acceleration of the measured motion is higher than the equivalent linear. The same last observation holds also for the spectrum calculated by NL-DYAS. The difference in the frequency content between the measured and the calculated spectra in conjunction with the agreement of the content between the spectra predicted by the two distinctive numerical methods seem to indicate that the soil deposit is possibly less stiff than what has been estimated by Stevens et al. (1999).

Therefore, taking into account the measured response spectrum hump at periods between 0.12 and 0.22 sec, a new shear wave velocity profile reduced by about 20% and described by the relation:

\[
V_s(z) = 130 \cdot z^{0.25}
\]

was used to run the analyses again. This modification of the velocity profile is justified regarding the difficulty to measure accurately the dynamic soil properties in the centrifuge as the influence of the soil container to the specimen response is hardly known (Boulanger, 2005). Moreover, such a modification has been used again in a similar experiment on the same centrifuge (Boulanger et al., 1999).
In Figs. 5 and 6 the results of the new analyses are depicted for centrifuge acceleration of 10 g. In this case the nonlinear acceleration time history compares rather well to the measured one while the motion predicted by the equivalent linear method still exhibits higher values (Fig. 5). In terms of response spectra (Fig. 6) NL-DYAS produces a spectrum almost identical to the measured one not only in PGA value but also in spectral values for the whole period range. On the other hand, the equivalent linear method, although it has the same frequency content with that of the experimental spectrum, overestimates systematically the acceleration values (PGA $\approx 1$ g instead of 0.5 g, and $S_{a_{\text{max}}} \approx 3.5$ g instead of 2 g).

Figure 3. Comparison of measured and calculated acceleration time histories for centrifuge acceleration of 10 g and shear wave velocity given by: $V(z) = 165 \ z^{0.25}$

Figure 4. Acceleration response spectra at the surface of the deposit for centrifuge acceleration of 10 g and shear wave velocity given by: $V(z) = 165 \ z^{0.25}$
In Fig. 7 the maximum acceleration, shear strain and shear stress profiles are shown as calculated by NL-DYAS and SHAKE. Especially for the maximum acceleration profile data from the centrifuge test are also illustrated. It is noticed that the nonlinear method compares well to the experimental results throughout the whole depth of the deposit. At the same time, it is obvious that equivalent linear method tends to overestimate the acceleration especially at few meters below the ground surface. The phenomenon could be attributed to the increased near the surface shear strain observed in SHAKE analysis due to the almost zero shear modulus at the same area. The same issue has been reported and investigated by Travasarou and Gazetas (2004).
Using the same dynamic soil properties and the modified shear wave velocity profile we conducted a new set of analyses for the case of centrifuge acceleration equal to 40 g (deposit depth: 24 m) and compared with the experimental results. Fig. 8 depicts the experimental and calculated acceleration response spectra. The aforementioned remarks hold here too: the BWGG model predicts in general accurately the site response while the equivalent linear model produces rather unreliable results.

CONCLUSIONS

A phenomenological constitutive model implemented into a finite differences computer code was presented in this paper and found capable of predicting efficiently the nonlinear site response. The few model parameters were calibrated using the well established $G$-$\gamma$, $\xi$-$\gamma$ curves and their validity was verified through comparison with experimental results. A quite good agreement was achieved, increasing in this way the confidence on the proposed model.

As reported by previous researchers (Lo Presti et al., 2006) and portrayed in this paper, the equivalent linear method tends to overestimate the ground response in cases of increased excitation magnitudes. Furthermore, equivalent linear model fails to accurately predict the site response when the shear modulus close to ground surface tends to zero values.

Finally, caution should be exhibited when using experimental measurements of shear wave velocities in specimens. The effect of the soil container on the deposit is usually unknown and this could result into misleading conclusions.
Figure 8. Acceleration response spectra at the surface of the deposit for centrifuge acceleration of 40 g and shear wave velocity given by: $V_0(z) = 130 z^{0.25}$

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