UNCERTAINTIES ASSESSMENT IN SURFACE WAVE TESTS FOR SITE RESPONSE STUDIES

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ABSTRACT

Surface wave tests are aimed at evaluating the small strain shear modulus profile at a site with an inversion process based on the modeling of geometrical dispersion of Rayleigh waves. One of the critical aspects of any geophysical test based on inversion processes is given by non-uniqueness of the solution. Indeed several combinations of model parameters can be associated to responses of the forward problem very similar and practically equivalent with respect to experimental data. The consequences of the non-uniqueness of the solution should be carefully evaluated with respect to the final objective of the survey in order to assess their relevance.

In the present study the case of seismic site response studies is considered. Shear wave velocity profiles are obtained from surface wave data using a Monte Carlo approach and a statistical test that allows the selection of a number of equivalent profiles within a certain level of confidence. With this approach it is possible to take into account both experimental uncertainties in the estimation of the dispersion curve and uncertainties deriving from non-uniqueness of the solution. These profiles are then independently used for evaluating the seismic site response. Two case histories related to sites in alpine valleys are reported. It is shown that equivalent profiles with respect to surface wave testing are equivalent also with respect to site amplification.

The results show that surface wave tests can be considered a reliable and effective tool for site response studies.

Keywords: Site Response, Surface Waves, SASW, Monte Carlo, Site Characterization

INTRODUCTION

The small strain shear modulus is strongly affected by disturbance induced by sampling and reconsolidation in laboratory estimates: it is therefore necessary to use in situ seismic methods to obtain a reasonable estimate of this parameter (Stokoe and Santamarina, 2000). Surface wave test are widely used for the determination of the small strain shear modulus profile in soils especially for seismic site response studies because they are cost and time effective options for testing large areas and do not suffer from many practical limitations that affect body wave techniques (SH refraction surveys).

Since surface wave tests are based on the solution of an inverse problem, the mayor criticism in their respect is related to the non-uniqueness of the solution and to the equivalence problem. Indeed several different profiles can be associated to numerical dispersion curves of Rayleigh waves having a good agreement with the experimental one. This causes uncertainty on the final profile adopted for the soil.

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Nevertheless it is worthwhile to consider that the dispersion curve represents the global dynamic behavior of the site. Moreover, the soil model adopted for the interpretation of surface wave data (a stack of homogeneous isotropic layers) is also used for several engineering applications. Hence, the equivalence problem is likely to have no severe consequences. E.g. if several soil profiles are equivalent in terms of surface wave propagation, they are likely to be equivalent also in terms of the seismic site response if it is evaluated using a 1D model. This aspect is systematically explored in the present work using a Monte Carlo approach. The inversion problem is solved exploring the space of the solution and selecting a set of profiles which, in statistical terms, are equivalent with respect to the experimental dispersion curve and its uncertainty. Then these profiles are used for numerical simulations of seismic site response in order to assess the influence of uncertainty and non-uniqueness of the inversion solution.

In the first section of the paper, some selected aspects of surface wave testing are presented, outlining the overall procedure and focusing in particular on the problem of equivalence, caused by non-uniqueness of inversion solution and on the Monte Carlo Inversion (MCI) algorithm used. Then the influence of equivalence on the final results of site amplification studies is assessed with reference to two case histories in Northern Italy.

SURFACE WAVE TESTS

Surface wave tests are based on the property of geometrical dispersion which makes the velocity of propagation of Rayleigh (and Love) waves dependent on frequency in vertically heterogeneous media. The experimental dispersion curve can be evaluated on the basis of field data using a variety of processing techniques (e.g., Foti, 2005; Socco and Strobbia, 2004). The experimental dispersion curve is then used for the solution of the inverse problem aimed at the identification of the parameters of the adopted model. In this respect, the soil is usually modeled as a stack of homogeneous linear elastic layers and the parameters to be identified are the layer thicknesses and shear wave velocities. Poisson ratios and density variations are usually neglected because of the small sensitivity of the Rayleigh wave dispersion curve to these parameters.

In general, it is possible to distinguish between active surface wave tests, in which the waves are on-purpose generated using a seismic source, and passive surface wave tests, which are based on the analysis of microtremors. Different acquisition setup and processing strategies can be used for the estimation of the experimental dispersion curve, often associated to different acronyms adopted for surface wave testing (SASW, MASW, CSW, SWM, REMI,…). This aspect causes some confusion on the topic, especially for the final users.

Once the experimental dispersion curve has been estimated, several strategies can be adopted to solve the inversion problem. In general, it is possible to classify the approaches in local search methods (e.g., trial and error approach or least squares algorithms) and global search methods (Monte Carlo approach, simulated annealing, genetic algorithms). Using one of these strategies, the shear wave velocity profile associated to a numerical dispersion curve as close as possible to the experimental dispersion curve is selected.

Equivalence (non-uniqueness of the solution)

One of the major criticisms for surface wave testing derives from the problem of non-uniqueness. Indeed, any inverse problem is ill-posed from the mathematical point of view, and it is not possible to demonstrate the uniqueness of the solution (Tarantola, 2005). In practice, several shear wave velocity profiles can be associated to numerical dispersion curves having similar distance from the experimental one. Taking into account the uncertainty in the measurement and estimation of the experimental dispersion curve, it is possible to identify several profiles which are statistically equivalent with respect to the experimental dispersion curve. This aspect becomes particularly critical when a deterministic approach is adopted for the inversion, since the final result becomes very
sensitive to the initial model and the inversion process can easily be biased by wrong choices in term of model parameterization that lead the solution into local minima. Obviously as any inverse problem, the inversion of surface waves can benefit from external constrains and a-priori information, which reduce the uncertainty of the solution.

Monte Carlo inversion
A global search method based on a Monte Carlo procedure (Boiero et al., 2006) has been used for the inversion in the present study to obtain a number of profiles which can be considered statistically equivalent with respect to the uncertainty in the experimental data. A population of $10^5$ synthetic models has been randomly generated after defining the number of layers and the starting upper and lower boundary for each model parameter. For each model, the fundamental mode curve is computed using the Transfer Matrix approach (Thomson, 1950; Haskell, 1953). The algorithm has been designed to be particularly efficient thanks to the introduction of scale properties of the dispersion curve.

The misfit function between experimental and numerical dispersion curve is then evaluated for each random profile, accounting for the data uncertainty and the statistical number of degrees of freedom. The misfit function considered in the present study is:

$$
\chi^2 = \sum_{n} (V_t - V_e)^2 \sigma_e^{-2}
$$

where $V_t$ and $V_e$ are respectively the theoretical and experimental phase velocities, $\sigma_e$ contains the standard deviations of the experimental phase velocities, $m$ is the number of points in the dispersion curve and $n$ is the number of layers above the half-space in the model.

The $\chi^2$ ratio is evaluated for each synthetic model. A Fisher test (Sachs, 1984) is then used to select the models that can be considered statistically equivalent for a given level of confidence. The result of the inversion is therefore a group of models having a misfit on the dispersion curve lower than a certain variable threshold, determined on the basis of the level of confidence (Boiero et al., 2006). The range of variation of each parameter could not be assumed as an uncertainty of the single parameter but it is obvious that well resolved parameters present small range of variations while badly resolved parameters can assume virtually any value in a wide range.

For the present study a level confidence equal to 1% has been used in both field cases: the result of the inversion is therefore a group of models that select a region in the model space within which the “true one” falls at 99% of probability according to the information present in the experimental data. With this approach it is possible to take into account both experimental uncertainties in the estimation of the dispersion curve and uncertainties deriving from non-uniqueness of the solution.

CASE HISTORIES

Torre Pellice
The site is located in Torre Pellice, a town in Val Pellice an alpine valley in Piedmont (Italy). The local geology presents an overburden mainly composed by a layer of fluvial sediments (compacted sands and gravels) with an expected thickness of 10 – 50 m over a lacustral sedimental layer (mainly composed by clays), down to the bedrock. The bedrock depth is expected to be more than 100 m in the central part of the valley.
**Site Characterization**

The site characterization program in Torre Pellice was based on both site and laboratory tests. Figure 1 reports the shear modulus and damping ratio decaying curves obtained with resonant column tests on the clayey layers. As the gravelly layers are concerned, considering the impossibility of retrieving undisturbed samples, the decaying curves have been taken from the literature (Seed et al., 1986).

As in situ tests are concerned a variety of seismic techniques have been used. In particular a combined P-wave refraction and reflection survey was conducted to locate the bedrock position. For the determination of the shear wave velocity profile, a Down-Hole test has been conducted to a depth of 40m. Also the data from the reflection-refraction survey have been processed to extract the experimental dispersion curve of Rayleigh waves. Using the large amount of available experimental data has been possible to estimate both the average values of phase velocities and their standard deviation by processing separately different shots (Socco et al., 2006).

![Figure 1. Decaying curves of clayey layers from Resonant Column tests.](image1)

**Figure 2. Experimental results at Torre Pellice site – Active Surface Wave tests:**
(a) Experimental and numerical dispersion curves; (b) Shear wave velocity profiles
This experimental dispersion curve has been interpreted using the MCI approach, adopting a confidence level of 1%. Hence the profiles reported in Figure 2b can be considered statistically equivalent with respect to surface wave propagation. Figure 2a shows all the numerical dispersion curves corresponding to the selected profiles (Figure 2b) compared to the experimental one, showing the equivalence of these profiles with respect to experimental data. The relative misfit representation adopted for both the equivalent profiles and corresponding dispersion curves shows the absolute difference between each profile misfit and the lowest misfit, so that the darkest color corresponds to the profile having the lowest misfit.

It is noticeable that a good resolution is attained for shallow layers, while, as expected, the uncertainty is higher for deeper layers. This is partially due to the fact that the resolution of surface wave inevitably decreases with depth and partially to the higher experimental uncertainty at low frequency (Figure 2).

Reference ground motion
The reference ground motion has been selected on the basis of probabilistic seismic hazard studies. In particular reference is made to the results available from a study conducted for the whole Italian territory (Naso et al., 2005) and to the simplified de-aggregation procedure proposed by Bommer et al. (2000).

From the above procedure, the reference earthquake for Torre Pellice site is given by a Magnitude 5.0 event at epicentral distance 9km, associated to a strike-slip rupture. Using this parameters, real accelerograms recorded on rock sites have been selected from the European Strong Motion Database (Ambraseys et al., 2002). These records have been scaled to the PGA for the site, which is equal to 0.14g for a return period of 475 years.

Results
The seismic site response has been assessed using the code Shake91 (Idriss and Sun, 1992). In particular the set of shear wave velocity profiles obtained from the MCI have been used for independent runs of the numerical analysis. The reference bedrock has been placed at 80m from the ground surface on the basis of the information retrieved from an independent combined reflection-refraction survey. The results of the numerical analysis for one of the selected real accelerogram are reported in Figures 3 and 4.

Figure 3. Amplification Functions at Torre Pellice site.
In particular Figure 3 reports the amplification function for the different profiles, while Figure 4 shows the response acceleration spectra for the different profiles which have been obtained from the MCI. The same color scale base on relative misfit is adopted, so that the darkest color corresponds to the profile having the lowest misfit in the surface wave inversion.

It can be observed that equivalent profiles in terms of surface wave inversion are equivalent also in terms of site amplification, giving very similar response spectra on the ground surface (the average coefficient of variation of the spectra is 2% with a maximum absolute value of 8.9%).

La Salle
The site is located in La Salle (Valle d’Aosta) on a wide fluvial fan, mainly composed by gravels and sands with some silt content. A borehole logged up to 50m from the ground surface for the execution of one of the Down-Hole tests reports layers of gravelly sands and silty sands with gravel.

Site Characterization
Undisturbed sampling was not feasible at this site because of the nature of the sediments; hence the decaying curves have been taken from the literature (Seed et al., 1986).

Also in this site a combined reflection-refraction survey has been conducted to locate the bedrock position. Surface wave data tests have been performed at this site using both active and passive methods (Foti et al., 2006). In this paper only the results from passive tests are reported in order to assess the consequences of lack of information in the high frequency range on the final results in terms of seismic site response and to compare the results with the previous test site where only active data were available. In particular the acquisition of microtremors has been carried using a 75m circular array with 12, 2Hz geophones equi-spaced along the circumference. Average values and standard deviation of the experimental dispersion curves have been obtained processing several different acquisitions of microtremors.

Figure 5a shows all the numerical dispersion curves corresponding to the selected profiles (reported in Figure 5b) compared to the experimental one, showing the equivalence of these profiles with respect
to surface wave data. The same relative misfit representation of the previous section has been used. Since passive data are concerned for this test site the resolution on shallow layers is not comparable to the one obtained for the previous test site. However as it can be noticed from figure 5b a clear cluster of low misfit profiles appears for deeper layers as in the potentiality of passive tests.

![Figure 5. Experimental results at La Salle site – Passive Surface Wave tests: (a) Experimental and numerical dispersion curves; (b) Shear wave velocity profiles](image)

**Reference ground motion**

The same procedure of the previous site has been adopted to select the reference ground motion. In particular the reference earthquake in this case is given by a Magnitude 5.5 event at epicentral distance 18km, associated to a normal fault rupture. The real accelerograms retrieved from the European Strong Motion database have been then scaled to the PGA for a return period of 475 years equal to 0.11g.

**Results**

Also in this case, the seismic site response has been assessed for each of the shear wave velocity profiles obtained from the Monte Carlo inversion. The reference bedrock has been placed at 100m from the ground surface on the basis of the information retrieved from the combined reflection-refraction survey. The results of the numerical analysis for one of the selected real accelerograms are reported in Figures 6 and 7.

The obtained results confirm the conclusion drawn from the previous case history. Despite the lack of information in the high frequency range and the consequent uncertainty in the identification of very shallow layers, typical of passive surface wave tests, the response spectra for the equivalent profiles are very similar. Hence also in this case equivalent profiles in terms of surface wave inversion are equivalent also in terms of site amplification. In particular, the average coefficient of variation of the acceleration spectra is 2.1% with a maximum absolute value of 9.4%.
CONCLUSIONS

A mayor criticism for surface wave methods, which represent cost and time effective tests for site characterization in seismic site response studies, is due to the equivalence problem, which arises from the non-uniqueness of the solution in the inversion. This aspect is particularly critical when no a-priori information on soil stratigraphy is available.

An assessment of the relevance of the equivalence problem should be always made with respect to the final objective of site characterization, in order to estimate the possible consequences in term of uncertainties on the final results. In the present paper two case histories of site amplification studies have been presented showing that the equivalence problem of surface wave testing, in spite of causing very often a high degree of uncertainty on model parameter estimation, is likely to cause only minor
differences in terms of the response spectra for the site in cases in which a 1D model is reasonable in the view of local geology. Moreover it as to be considered that other sources of uncertainties in the assessment of seismic site response have not been considered in the present work (e.g. uncertainties in the estimation of soil non-linearity or uncertainties in the choose of the reference seismic motion). The obtained results confirm surface wave tests as a very valuable tool for site amplification studies.

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